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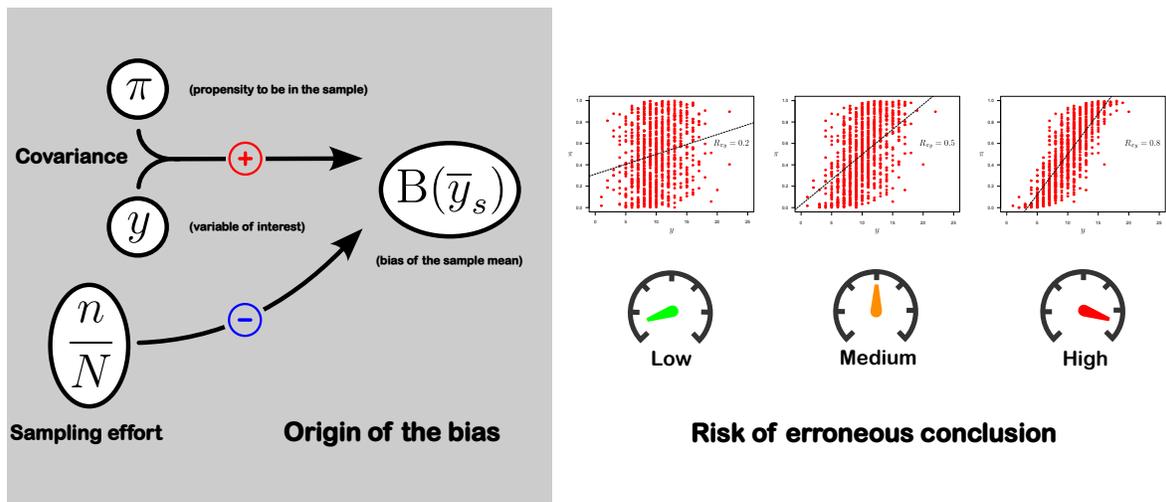
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Graphical Abstract

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Highlights

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- Preferential sampling can be modeled by Poisson or conditional Poisson sampling.
- We document the impact of preferential sampling on population mean estimation.
- Preferential sampling can lead to biased mean estimation, if not accounted for.
- The bias increases with the covariance between sample membership and the variable.
- The bias decreases with increasing sampling effort (expected sampling fraction).

On the impact of preferential sampling on ecological status and trend assessment

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Abstract

Assessments of the status and trends of abiotic and biotic indicators are two central objectives in many ecological studies and monitoring programs. Given the impracticality of making measurements or observations at every point in geographic space, even within a limited domain, consideration of spatial sampling is crucial to ensure the reliability of statistical inference regarding such status or temporal trends.

The sampling units in geographic space (e.g., sites, plots, quadrats) for field observations are often selected with a preference for those expected to be species-rich or those with the highest abundances or occupancy probabilities. This sampling approach, called *preferential sampling*, can be based on probability sampling theory, but in practice, it is usually a form of nonprobability sampling.

Introducing a selection force that disproportionately includes units in the sample based on the expected values of the variables of interest can lead to (severely) biased inferences. This is because inclusion probabilities — referred to here as *propensities* for units to be part of the sample — cannot be accounted for in statistical estimators when they are unknown to the sampler.

In this article, we model sampling processes (considered without replacement) for a finite spatial population of sampling units using probability sampling designs. We consider four designs: Bernoulli sampling, Poisson sampling, simple random sampling, and conditional Poisson sampling. We document the bias introduced by preferential sampling in the estimation of a mean, whether for a status assessment (e.g., mean species richness) or a trend assessment (e.g., trend in mean abundance). For this purpose, we use Monte Carlo simulations and an analytical expression for the bias of the sample mean.

This analytical expression shows that the bias of the sample mean (1) increases with increasing covariance between the propensities and the values of the variable of interest and (2) decreases with increasing sampling effort (sampling fraction or expected sampling fraction). This fundamental statistical result is neither widely known nor appreciated by most ecologists, even though it has the potential to ruin status or trend assessments and to lead to erroneous conclusions.

The findings on preferential sampling in ecology presented in this article are reviewed from a methodological perspective, mainly for an audience of quantitative ecologists, wildlife statisticians, and biometricians involved in the design or implementation of ecological studies and monitoring programs. To facilitate future exchange among researchers on this topic by clarifying the concepts, in the discussion we also examine the terminology found in the literature for the notions related to preferential sampling.

Keywords:

biased site selection, spatial sampling processes, propensities, range shift, abundance gradient, conditional Poisson sampling

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1. Introduction

Although it has been abandoned in modern physics, the separation of space and time proves convenient and operational at macroscopic scales relevant to ecology. This allows us to consider all ecological phenomena as occurring in space and time, whether these dimensions are considered jointly or separately in their description, analysis and modeling.

In practice, the measurements or observations made, whether on abiotic or biotic variables, concern a given spatial domain (which we denote \mathcal{D}) and which take place over a given period (which we denote \mathcal{T}). Even if \mathcal{D} and \mathcal{T} are not large, it is generally impossible to make measurements or observations at every point in \mathcal{D} or every time in \mathcal{T} . Thus, a fundamental sampling problem immediately arises.

In the following, we assume that \mathcal{D} is a two-dimensional spatial domain lying in Euclidean space. Although not mandatory, \mathcal{D} is often partitioned into a finite population of areal units (see, e.g., [Aubry and Francesiaz, 2022](#), Fig. 1). The same is true for the time domain, which can be discretized into a finite population of time units (e.g., potential daily counting sessions for bird counting).

Regardless of whether the population of spatiotemporal units is finite or considered as infinite, data collection is performed in two steps: (i) by selecting a subset of units from $\mathcal{D} \times \mathcal{T}$ to form a sample s and (ii) by making measurements or observations on the units of s .

In the technical mathematical sense, an *error* is the deviation between a state (or a summary of a state) defined on $\mathcal{D} \times \mathcal{T}$ and its estimate or prediction computed from the sample s at hand. The first step mentioned above introduces a sampling error because only a portion of the sampled spatiotemporal population is considered when collecting the data. The second step generates an error that can be treated as a measurement or observation error, for instance, when counting individuals (e.g., [Aubry et al., 2012](#)) or when assessing the size of vast groups of wild animals (e.g., [Vallecillo et al., 2021](#)). When individuals are counted, this error can also be treated as a sampling error — where the set of individuals counted is a sample of all individuals present — due to *imperfect detection* (e.g., [White, 2005](#); [Kellner and Swihart, 2014](#); [Perret et al., 2023](#); [Johnston et al., 2023](#), Sec. 4.2). In this article, we will focus only on the first source of error, concentrating on the sampling of \mathcal{D} . Since we are interested only in spatial sampling, we do not discuss methods for counting individuals or problems of imperfect detection or of imperfect ability to identify and to count them when they are detected (see, e.g., the references cited by [Vallecillo et al., 2021](#) and [Aubry et al., 2023](#)).

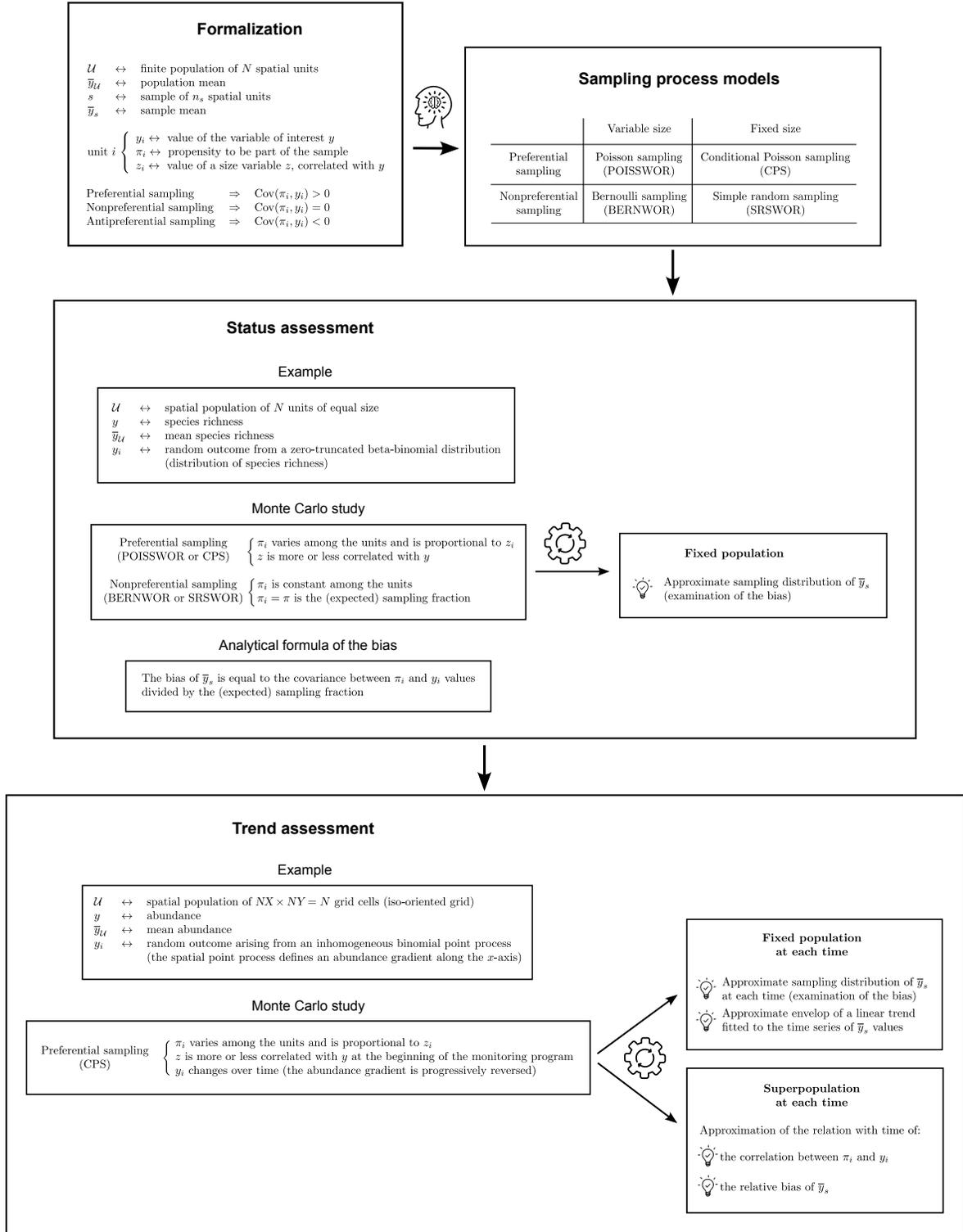
In a general sense, we call the *sampling process* the way by which a sample s is selected from a statistical population, regardless of the method used. We reduce the set of sampling processes by assuming here that they are without replacement. We assume that the sampling process under consideration can be replicated — at least in principle — so that distinct samples (see [Hedayat and Sinha, 1991](#), p. 2) can be obtained on the basis of selection probabilities $0 < p(s) < 1$. The individuals or groups of individuals who perform the sampling may be collectively referred to as the *sampler*. The sampling processes considered in this article are therefore human-based and, as such, are not necessarily easy to model in detail (but see [ter Steege et al., 2011](#); [Fernández and Nakamura, 2015](#)).

Statistical inference from a sample can be used to estimate parameters of an actual finite statistical population or parameters of a hypothetical superpopulation describing the stochastic process under study (see, e.g., [Aubry and Francesiaz, 2022](#), Sec. 2.3). Sampling is said to be *preferential* when the sampling process is related to a superpopulation (e.g., [Diggle and Ribeiro, 2007](#), Sec. 4.4.2; [Diggle et al., 2010](#); [Watson et al., 2019](#); [Gray and Evangelou, 2023](#)) or to the finite population at hand in such a way that units are more likely to be included in the sample if they have the highest values for the indicator of interest. For example, in ecology, preferential sampling may refer to preferential inclusion in the sample of the most species-rich units, the units with the highest abundances or occupancy probabilities. Conversely, we use the expression *antipreferential sampling* to denote situations where units with the lowest values are more likely to be included in the sample. In all the other cases where the probability of inclusion in the sample is independent of the characteristics of the unit with respect to the variables of interest, sampling is said to be nonpreferential.

With the previous definition in mind, preferential sampling should be distinguished from (i) *convenience sampling* (sometimes called *accessibility sampling*; see, e.g., [Young and Young, 1998](#), p. 93 or [Barnett, 2002](#), p. 17) and (ii) *purposive sampling* (also known as *judgment sampling*). In the first case, units are selected for inclusion in the sample for convenience (e.g., ease of access and/or safety for the observers). In the second case, units judged to be typical or appropriate for the survey are selected from the statistical population under study, for example, when "ecologists actively seek the event of interest, such as an active breeding nest of a bird, or a specific plant species" ([Edwards et al., 2006](#)). The previous two sampling methods correspond to two forms of nonprobability sampling. Preferential sampling can be either a nonprobability sampling method or be based on a random selection mechanism. We distinguish between the two cases (i.e., nonprobability vs. probability preferential sampling) because the statistical implications are quite different. Indeed, in the first case, statistical inference

61 must be based on a model — which can be implicit — and its key assumptions, while in the second
62 case, the properties of the estimator used can be based solely on the random selection mechanism (see,
63 e.g., [Aubry and Francesiaz, 2022](#)).

64 As a concrete situation, we assume in this article that sampling is performed in a nonprobability
65 setting. We consider two objectives that are central to many ecological studies or monitoring programs:
66 assessment of (i) status (i.e., the state observed at a given point in time) or (ii) trend (i.e., a smooth
67 pattern of state variation over time) (see, e.g., [Gitzen et al., 2012](#)). We do not discuss the scientific
68 relevance of either of these depending on the context in which they occur (see, e.g., [Vos et al., 2000](#);
69 [Yoccoz et al., 2001, 2003](#); [Nichols and Williams, 2006](#)). We merely consider status and trend as
70 objects of statistical inference, and our aim is to examine the impact of (nonprobability) preferential
71 (spatial) sampling on their assessment, mainly in terms of estimation bias. To this end (i) we formalize
72 preferential sampling in the context of finite population sampling; (ii) we briefly consider the one-stage
73 probability sampling designs that can be used to model basic without-replacement sampling processes;
74 (iii) using appropriate sampling process models, we examine the case of status and trend estimation
75 under preferential sampling through Monte Carlo simulation studies; (iv) we introduce to ecology a
76 fundamental formula that enables us to understand the very nature of the mean estimation bias that
77 can be caused by preferential sampling. Finally, we draw lessons from the results presented, particularly
78 with respect to trend assessment. [Fig. 1](#) is intended to provide the reader with an overview of the
79 article organization and of the topics covered before the discussion and perspectives sections.



Trend assessment

Example

\mathcal{U} \leftrightarrow spatial population of $N_X \times N_Y = N$ grid cells (iso-oriented grid)
 y \leftrightarrow abundance
 $\bar{y}_{\mathcal{U}}$ \leftrightarrow mean abundance
 y_i \leftrightarrow random outcome arising from an inhomogeneous binomial point process (the spatial point process defines an abundance gradient along the x -axis)

Monte Carlo study

Preferential sampling (CPS) $\left\{ \begin{array}{l} \pi_i \text{ varies among the units and is proportional to } z_i \\ z \text{ is more or less correlated with } y \text{ at the beginning of the monitoring program} \\ y_i \text{ changes over time (the abundance gradient is progressively reversed)} \end{array} \right.$

Fixed population at each time

Approximate sampling distribution of \bar{y}_s at each time (examination of the bias)
 Approximate envelop of a linear trend fitted to the time series of \bar{y}_s values

Superpopulation at each time

Approximation of the relation with time of:

- the correlation between π_i and y_i
- the relative bias of \bar{y}_s

Figure 1: Overview of the article organization before the discussion and perspectives sections. We first formalize preferential sampling in the context of finite population sampling. Next, we consider the probability sampling designs that can be used to model basic nonpreferential/preferential sampling processes. With these models, we then use Monte Carlo simulations to study the impact of preferential sampling, first for an example of status assessment, second for an example of trend assessment. The type of results to be examined is indicated to the right, either in a fixed population context or in a superpopulation context.

80 2. Formalization

81 From a population of spatial units (e.g., sites, plots, quadrats) \mathcal{U} of size N , spatial sampling consists
82 of selecting a subset $s \subseteq \mathcal{U}$ of size $n_s \leq N$, called a sample. The sampled spatial population should
83 not be confused with a biological population, i.e., here a set of organisms present in the domain \mathcal{D} .

84 The size of the spatial population N is not necessarily known to the sampler. In addition, the
85 sample size n_s may be variable or fixed. It is variable, for example, when the sample is drawn by a
86 group of people who do not necessarily consult each other and who each add one or more units to
87 the sample. The sample size can be fixed if the sample selection is more centralized, for example, to
88 predetermine the sampling effort to be allocated.

89 In the following, we consider a univariate situation, i.e., there is only one variable of interest y ,
90 which takes fixed values for the units $i \in \mathcal{U}$. We denote $\mathbf{y} = (y_1, y_2, \dots, y_N)$ as the vector of y -values for
91 $i \in \mathcal{U}$. By extension, this vector can be called the population (implicitly of y -values) associated with the
92 population of spatial sampling units \mathcal{U} . The \mathbf{y} -vector results from a bioecological stochastic process,
93 which may be modeled if appropriate. The observed \mathbf{y} -vector is then assumed to be a realization of a
94 random vector (i.e., a superpopulation).

95 2.1. Inclusion probabilities

96 In any sampling process, the probability $0 < \pi_i \leq 1$ that a unit $i \in \mathcal{U}$ is part of s : (i) is a probability
97 of inclusion known to the sampler, as is the case in finite population sampling theory (e.g., [Särndal
98 et al., 1992](#); [Tillé, 2020](#)); or (ii) expresses the greater or lesser propensity of a unit to be part of the
99 sample, which is unknown to the sampler. In the latter case, the probabilities π_i ($i \in \mathcal{U}$) are *propensity
100 scores*, here simply called *propensities*. We denote $\boldsymbol{\pi} = (\pi_1, \pi_2, \dots, \pi_N)$ as the vector of probabilities
101 for $i \in \mathcal{U}$. Let us note that the case $\pi_i = 0$ corresponds to the exclusion of unit i from the sampled
102 population, a trivial case not of interest here.

103 Whether we are dealing with inclusion probabilities in the strict sense or with propensities, the $\boldsymbol{\pi}$ -
104 vector results from the sampler's view of the variation of the variable of interest across geographic space,
105 without actually knowing it precisely. In particular, the probabilities considered in this article are not
106 modified when y -values are observed; therefore, the sampling process can be considered nonadaptive.
107 Thus, in this article, we assume that the samples are generated by a sampling process governed by
108 probabilities that are fixed in advance, whether consciously or not.

109 Where appropriate, the probabilities π_i can be defined as being proportional to a variable z ($z_i > 0$
110 for all $i \in \mathcal{U}$), which assigns more or less importance to the units — this type of variable is known as
111 a *size variable* — and is assumed to be correlated to some level with the variable of interest y .

112 2.2. Preferential sampling

113 The sampling process is said to be preferential when there is a monotonically increasing relation
114 between the y -values and the inclusion probabilities, whether the latter are known or unknown. If the
115 relation decreases, we have antipreferential sampling.

116 We do not assume that the relation is linear, as this assumption is too restrictive to be realistic.
117 Indeed, a linear relation assumes that the propensity increases at a constant rate as a function of the
118 y -variable, whereas in reality, it may increase faster at higher y -values, for example. Moreover, the
119 relation in question is not necessarily analytical; it can be only statistical — corresponding to the
120 conditional expectation — if the propensity varies for each y -value.

121 A necessary condition for sampling to be preferential (or antipreferential) is that the probabilities
122 π_i ($i \in \mathcal{U}$) differ significantly between units or at least between groups of units. Sampling is inherently
123 nonpreferential if all units have, at least approximately, the same probability of being included in the
124 sample.

125 2.3. Notation

126 In any sampling problem, we can distinguish at least two sources of stochasticity (see, e.g., [Aubry
127 and Francesiaz, 2022](#)): (i) one that relates only to the sampling process $p(s)$ of the finite population
128 of units, resulting in the $\boldsymbol{\pi}$ -vector and (ii) the other related to the stochastic process ξ at the origin
129 of the \mathbf{y} -vector (superpopulation). To avoid any ambiguity, operators (expectation, variance, etc.) are
130 denoted by using p as a subscript for the first source and ξ for the second. For a parameter ω , the
131 sampling distribution of an estimator $\hat{\omega}$, i.e., the distribution of all the values that $\hat{\omega}$ can take for
132 all the samples that can be generated, is called the p -distribution. The mean of the p -distribution is
133 called the p -expectation of $\hat{\omega}$, its variance is the p -variance (i.e., the sampling variance), and so on.
134 We define the p -bias as $B_p(\hat{\omega}) = E_p(\hat{\omega}) - \omega$ and the relative p -bias as $B_p(\hat{\omega})/\omega$. For the stochastic
135 process ξ , we similarly denote the ξ -expectation, ξ -variance, and so on. When we consider both

136 sources of stochasticity at the same time, we associate the two subscripts, and we write, for example,
 137 ξp -expectation. Let us recall that in this article, in the context of double stochasticity, $\boldsymbol{\pi}$ is fixed, while
 138 \mathbf{y} is random. Other notations used in this article are detailed in [Appendix A](#).

139 2.4. Parameters and estimators

140 Regardless of whether the spatial population size N is known to the sampler, for a variable of
 141 interest y (discrete, including binary, or continuous), we assume that the parameter to be estimated
 142 is the population mean $\bar{y}_{\mathcal{U}}$.

143 In the following, we consider a concrete situation in which (a) the size of the spatial population
 144 is unknown to the sampler (there is no sampling frame) and (b) the inclusion probabilities are also
 145 unknown to the sampler (the sample is not the result of applying a probability sampling design). In
 146 the absence of auxiliary variables that would be (strongly) correlated with y and exhaustively known
 147 on \mathcal{D} , the only data available are the y_i values for $i \in s$ and the sample size n_s . Thus, by default, the
 148 population mean can be estimated by the sample mean \bar{y}_s , and the p -variance of \bar{y}_s can be estimated
 149 using:

$$\widehat{V}_p(\bar{y}_s) = \frac{s_y^2}{n_s} \quad (1)$$

150 3. Sampling process models

151 The fact that only a part (i.e., the sample) of the population of spatial sampling units is considered
 152 for data collection represents a source of uncertainty that may be modeled. This can be referred to as
 153 a *sampling model* ([Hobbs and Hooten, 2015](#), Sec. 1.1.3) or an *inclusion model* ([Gelman et al., 2014](#),
 154 Sec. 8.2), although both terms are polysemous which can lead to confusion.

155 As noted by [Hájek \(1964, p. 1492\)](#), any probability distribution $p(s)$ can be used as a mathematical
 156 model for any sampling procedure, experiment, or method. Thus, in this article, we model a sampling
 157 process using an appropriate probability sampling design ([Aubry, 2023](#), Sec. 1.2.4). Basic sampling
 158 processes (without replacement) can be modeled by one of four one-stage sampling designs, described
 159 below, depending on whether the propensities are equal or unequal and on whether the sample size is
 160 fixed or variable (Fig. 2).

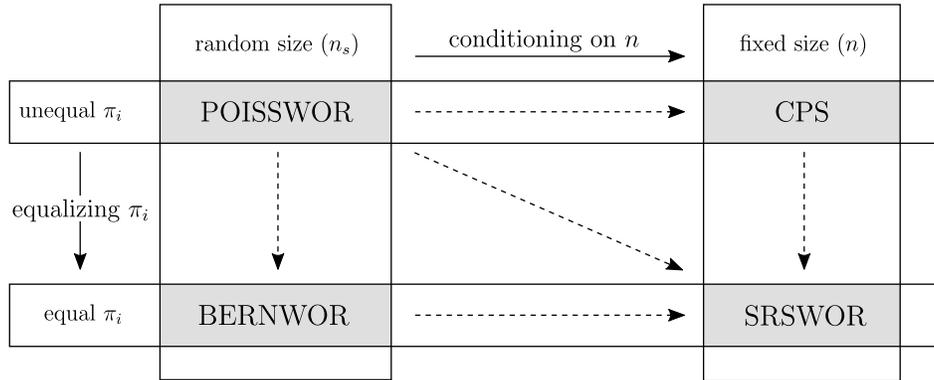


Figure 2: The four one-stage probability sampling designs used as without-replacement sampling process models. POISSWOR: Poisson sampling. BERNWOR: Bernoulli sampling. CPS: Conditional Poisson sampling. SRSWOR: Simple random sampling. The sampling designs are derived from each other by asymmetric relations of equalization of inclusion probabilities π_i ($i \in \mathcal{U}$) or conditioning on a fixed sample size n .

161 3.1. Preferential sampling — variable size (POISSWOR)

162 When the sample size is variable and the propensities vary across sampling units, the sampling
 163 process can be modeled by Poisson sampling, which we denote as POISSWOR.

164 In Poisson sampling, (i) the inclusion of a unit $i \in \mathcal{U}$ in sample s is governed by a Bernoulli trial
 165 with probability π_i and (ii) the resulting random sample size n_s follows a Poisson binomial distribution
 166 ($n_s \sim \text{PoiBin}(N, \boldsymbol{\pi})$) ([Hájek, 1981](#), Ch. 6; [Särndal et al., 1992](#), Sec. 3.5; [Tillé, 2006](#), Sec. 5.5; [Tillé, 2020](#), Sec. 5.8). Let us note that the Poisson binomial distribution is here the generalization of the
 168 binomial distribution for Bernoulli trials with unequal probabilities (see, e.g., [Wang, 1993](#)) and should
 169 not be confused with the compound distribution described by [Johnson et al. \(2005, Sec. 9.5\)](#).

170 A necessary condition for using POISSWOR as a model for variable-size sampling processes with-
 171 out replacement is that unit inclusions must be assumed to result from independent events. This
 172 assumption is broadly consistent with the situation in which the sample results from the actions of
 173 many people who do not coordinate with each other.

174 3.2. Nonpreferential sampling — variable size (BERNWOR)

175 When the sample size is variable and the propensities are sufficiently similar to each other such
 176 that they can be considered equal to a constant probability π , the sampling process can be modeled
 177 by Bernoulli sampling (also known as *binomial sampling*), which we denote as BERNWOR.

178 In Bernoulli sampling, (i) the inclusion of any unit in the sample is governed by a Bernoulli
 179 trial with probability π and (ii) the resulting variable sample size n_s follows a binomial distribution
 180 ($n_s \sim \text{Bin}(N, \pi)$) (Särndal et al., 1992, Sec. 3.2; Tillé, 2006, Sec. 4.3; Tillé, 2020, Sec. 3.2).

181 BERNWOR is a special case of POISSWOR when the probabilities π_i ($i \in \mathcal{U}$) are equal (Fig. 2).
 182 When modeling a sampling process, BERNWOR has the same independence condition as POISSWOR.

183 3.3. Preferential sampling — fixed size (CPS)

184 Conditional on a fixed sample size ($n_s = n$ for all s), POISSWOR becomes conditional Poisson
 185 sampling, abbreviated as CPS (Hájek, 1981, Ch. 14; Tillé, 2006, Sec. 5.6; Tillé, 2020, Sec. 5.9) (Fig.
 186 2). This is a general model suitable for a fixed-size preferential sampling process.

187 3.4. Nonpreferential sampling — fixed size (SRSWOR)

188 Conditional on a fixed sample size ($n_s = n$ for all s), BERNWOR becomes simple random sampling
 189 without replacement, abbreviated as SRSWOR, for which $\pi = n/N$ (Särndal et al., 1992, Sec. 3.3.1;
 190 Tillé, 2006, Sec. 4.4; Tillé, 2020, Sec. 3.3) (Fig. 2). SRSWOR is also a special case of CPS when the
 191 probabilities π_i ($i \in \mathcal{U}$) are equal (Fig. 2).

192 Let us recall that in this article, we assume that N is unknown to the sampler, so the propensity π is
 193 also unknown. SRSWOR is an appropriate model for a fixed-size sampling process with approximately
 194 constant propensities.

195 3.5. Estimators

196 Unlike the concrete situation we considered in Section 2.4, in Monte Carlo simulations, the π -vector
 197 is known, as is the population size N . In this case, the population mean can be estimated without
 198 p -bias using the expansion estimator for the population total (Horvitz-Thompson estimator; see, e.g.,
 199 Hedayat and Sinha, 1991, Ch. 3) to form the following weighted estimator:

$$\bar{y}_\pi = \frac{1}{N} \sum_{i \in s} \frac{y_i}{\pi_i} \quad (2)$$

200 For BERNWOR and SRSWOR, the estimator (2) simplifies to:

$$\bar{y}_\pi = \frac{1}{N} \frac{1}{\pi} \sum_{i \in s} y_i \quad (3)$$

201 which for SRSWOR gives the identity $\bar{y}_\pi = \bar{y}_s$ since, in this case, we have $\pi = n/N$.

202 For sampling processes modeled by BERNWOR or POISSWOR (variable sample sizes), the units are
 203 included in the sample independently. This gives a fairly simple expression for the p -unbiased estimator
 204 of the sampling variance (p -variance) of the weighted estimator (2) (see Särndal et al., 1992, p. 289).

205 The expression for POISSWOR is written as:

$$\hat{V}_p(\bar{y}_\pi) = \frac{1}{N^2} \left[\sum_{i \in s} \frac{1 - \pi_i}{\pi_i^2} y_i^2 \right] \quad (4)$$

206 which simplifies for BERNWOR to:

$$\hat{V}_p(\bar{y}_\pi) = \frac{1}{N^2} \left[\frac{1 - \pi}{\pi^2} \sum_{i \in s} y_i^2 \right] \quad (5)$$

207 In the case of fixed-size designs, in general, to estimate without bias the p -variance, it is necessary to
 208 know the joint probabilities $\pi_{ij} > 0$ that two units $i \neq j \in \mathcal{U}$ are simultaneously included in the sample.
 209 The general expression for the p -unbiased estimator of the p -variance of the weighted estimator (2)

210 for an unequal-probability, fixed-size, without-replacement sampling design such as the CPS is (see
 211 Sen-Yates-Grundy estimator, [Hedayat and Sinha, 1991](#), p. 52, Eq. 3.16):

$$\widehat{V}_p(\bar{y}_\pi) = \frac{1}{N^2} \left[\sum_{(i < j) \in s} \sum \frac{\pi_i \pi_j - \pi_{ij}}{\pi_{ij}} \left(\frac{y_i}{\pi_i} - \frac{y_j}{\pi_j} \right)^2 \right] \quad (6)$$

212 which simplifies for SRSWOR since then we have $\pi_i = n/N$ for all $i \in \mathcal{U}$ and $\pi_{ij} = n(n-1)/[N(N-1)]$
 213 for all $i \neq j \in \mathcal{U}$. After simplification, we obtain (e.g., [Tillé, 2020](#), p. 29, Result 3.1):

$$\widehat{V}_p(\bar{y}_\pi) = (1 - \pi) \frac{s_y^2}{n} \quad (7)$$

214 In the class of fixed-size without-replacement sampling designs with inclusion probabilities π , the CPS
 215 is the sampling design of maximum entropy ([Hájek, 1981](#); [Tillé, 2006](#); [Tillé, 2020](#), Sec. 5.9). It follows
 216 that an approximation to the p -variance estimator (6) may be obtained without involving the joint
 217 inclusion probabilities π_{ij} ($i \neq j \in \mathcal{U}$). We refer the reader to [Tillé \(2020, Sec. 5.14\)](#) and to the
 218 references cited by [Aubry \(2023, Sec. 1.3.2\)](#).

219 4. Status assessment

220 From an operational perspective, for a given spatial domain \mathcal{D} , status at a given point in time
 221 corresponds to either (i) the state of a variable of interest or (ii) the value of an indicator summarizing
 222 the state of the variable of interest. In practice, the point in time is actually a time interval — ideally,
 223 the shortest possible — which we have denoted \mathcal{T} . In the case of abundance, for example, at a certain
 224 infra-annual time interval \mathcal{T} over which a biological population may be assumed to be (approximately)
 225 closed both geographically and demographically, status in sense (i) may correspond to the spatial
 226 distribution of abundance, while in sense (ii), it may refer to average abundance. In the following, we
 227 refer to status in sense (ii) (indicator of interest).

228 4.1. Status example

229 As an example of a status to be estimated over a $\mathcal{D} \times \mathcal{T}$ spatiotemporal domain, we consider here
 230 the average species richness for a group of L species and a finite statistical population \mathcal{U} of N equal-size
 231 spatial sampling units, discretizing all or a part of the spatial domain \mathcal{D} . The variable of interest y is
 232 therefore species richness, and the indicator is the finite population mean $\bar{y}_\mathcal{U}$ (the time interval \mathcal{T} is
 233 implicit and omitted from the notation).

234 In this example, spatial units are considered habitat patches that are disjoint within the spatial
 235 domain of interest ([Wiens, 1976](#); [Hall et al., 1997](#); [Girvetz and Greco, 2007](#)). A habitat patch may
 236 be included in the sample from a previously established list of all patches (sampling frame) that has
 237 been established by habitat mapping, in which case N is known. In the absence of a sampling frame,
 238 habitat patches are included in the sample based on the sampler’s prior knowledge or on what the
 239 sampler encounters in the field, in which case N is generally unknown. This second case corresponds
 240 to the concrete situation of preferential sampling considered in this article (Section 2.4).

241 In the case of (inherently) nonpreferential sampling, units are selected with the same propensity
 242 π . With preferential sampling, units are selected based on a subjective assessment of their expected
 243 species richness, with propensities more or less positively correlated with y -values.

244 4.2. Population model example

245 As an illustrative example of a variable of interest y , we consider species richness with values
 246 ranging from 1 to $L = 25$, distributed according to a zero-truncated beta-binomial distribution
 247 $\text{BetaBinZT}(L, \alpha, \beta)$ with shape parameters $\alpha = 1$ and $\beta = 5$. These values are realistic since they
 248 are of the same order of magnitude as those obtained for the species richness of a set of 25 waterbird
 249 species (ducks, geese, swans, coots, waders and grebes) breeding in European France (see the LIMAT
 250 scheme described in [Aubry et al., 2023, Fig. 1](#)). According to the typology proposed by [Aubry and](#)
 251 [Francesiaz \(2022, Section 2.4, Table 1\)](#), this statistical distribution is a type IV superpopulation model;
 252 that is, it does not include information on spatial structure or covariates (auxiliary variables). This
 253 model specifies an infinite set of values, but we are interested here in sampling a finite population
 254 containing $N = 5000$ spatial sampling units.

255 Among the infinite set of populations containing $N = 5000$ y -values following a $\text{BetaBinZT}(25, 1, 5)$
 256 distribution, we choose one such that $|\hat{\alpha} - \alpha| < 10^{-3}$ and $|\hat{\beta} - \beta| < 10^{-3}$. The estimator used for the
 257 shape parameters α and β of the zero-truncated beta-binomial distribution is the second estimator

258 proposed by [Tripathi et al. \(1994, Sec. 3.1\)](#). We can visually check that the histogram of the variable
 259 y associated with the finite population \mathcal{U} we used is very close to that of the superpopulation model
 260 (Fig. 3). Thus, we have $\bar{y}_{\mathcal{U}} = 5.0062$ for comparison with its expected value in the model $E_{\xi}(\bar{y}_{\mathcal{U}}) = 5$.

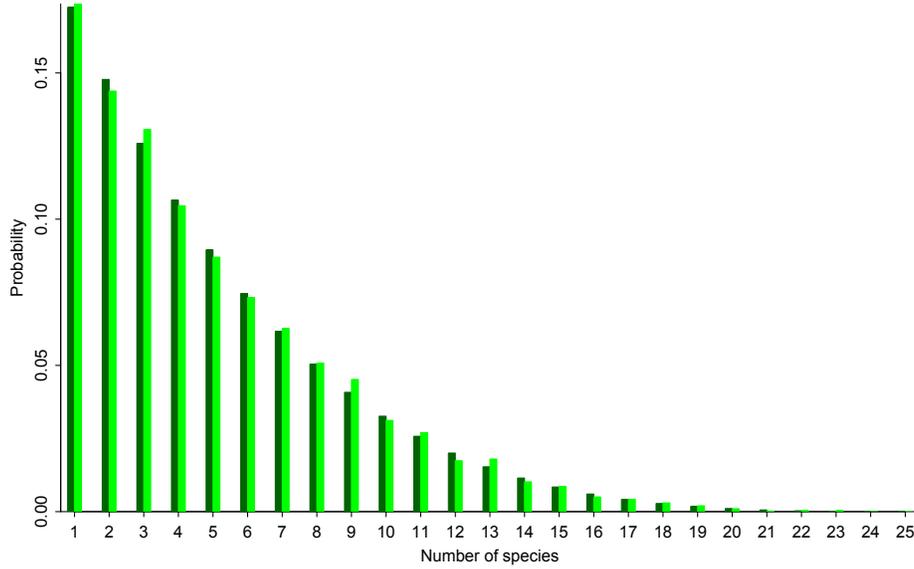


Figure 3: Distribution of the superpopulation model (in dark green) and histogram of the simulated population for $N = 5000$ (in light green).

261 4.3. Monte Carlo study

262 In this section, we examine the estimation of $\bar{y}_{\mathcal{U}}$ by \bar{y}_s in four situations, combining the preferential
 263 vs. nonpreferential nature of the sampling process and a variable vs. fixed sample size.

264 In the case of preferential sampling, the propensities are defined as being proportional to a size
 265 variable z ($z_i > 0$ for all $i \in \mathcal{U}$):

$$\pi_i = P \frac{z_i}{Z} \quad \text{with} \quad P = \sum_{i \in \mathcal{U}} \pi_i \quad \text{and} \quad Z = \sum_{i \in \mathcal{U}} z_i \quad (8)$$

266 In the case of POISSWOR or BERNWOR, we have $P = E_p(n_s)$ (variable sample size); in the case of
 267 CPS or SRSWOR, we have $P = n$ (fixed sample size).

268 Since we must respect the constraint $\pi_i \leq 1$, it follows that if we have $z_i > Z/P$ for at least one
 269 unit, we must perform the preprocessing described, for example, by [Aubry \(2023, Sec. 3.1\)](#) (see also
 270 [Tillé, 2020, Sec. 5.2](#)). From the above, we deduce that $R_{\pi y} = R_{zy}$ if and only if $z_i \leq Z/P$ for all $i \in \mathcal{U}$.
 271 In the following, we consider cases where this condition holds (for a counterexample, see [Aubry et al.,](#)
 272 [2020, Sec. 4.4](#)).

273 We define a model for the size variable z in which the correlation with y is ρ_{zy} . We vary ρ_{zy}
 274 between 0 and 1 by attenuating perfect positive correlation — for correlation attenuation; see, e.g.,
 275 [Charles \(2005\)](#) — by adding a Gaussian error term $\epsilon \sim N(0, \sigma_{\epsilon}^2)$ to y . Thus, for each $i \in \mathcal{U}$:

$$\begin{aligned} z_i &= \epsilon_i & \text{with } \sigma_{\epsilon}^2 &> 0 & \text{for } \rho_{zy} &= 0 \\ z_i &= y_i + \epsilon_i & \text{with } \sigma_{\epsilon}^2 &= S_y^2(1 - \rho_{zy}^2)/\rho_{zy}^2 & \text{for } 0 < \rho_{zy} < 1 \\ z_i &= y_i & \text{with } \sigma_{\epsilon}^2 &= 0 & \text{for } \rho_{zy} &= 1 \end{aligned} \quad (9)$$

276 Ultimately, to guarantee $z_i > 0$ (and hence, $\pi_i > 0$) for all $i \in \mathcal{U}$, we apply the linear transformation
 277 $z_i - \min(\mathbf{z}) + 0.1$.

278 In the same spirit as obtaining \mathbf{y} , we want the value of the finite population correlation R_{zy}
 279 to approximately match the superpopulation correlation ρ_{zy} . To achieve this, we generate random
 280 outcomes of the \mathbf{z} -vector until we obtain $|R_{zy} - \rho_{zy}| < 10^{-6}$. Having obtained the \mathbf{z} -vector, we then
 281 set a value for P and compute the $\boldsymbol{\pi}$ -vector according to expression (8).

282 In the following, we replicate a given sampling process model with $P = 500$, i.e., with 10% as the
 283 sampling fraction (fixed sample size) or expectation of the sampling fraction (variable sample size).
 284 For each replication, we compute \bar{y}_s and $\widehat{V}_p(\bar{y}_s)$. We use 10^6 replications to accurately approximate
 285 the p -distributions of these two estimators. Therefore, we can assess the possible estimation bias of

286 the population mean \bar{y}_U by the sample mean \bar{y}_s and also that of the p -variance $V_p(\bar{y}_s)$ by the default
 287 estimator $\hat{V}_p(\bar{y}_s)$ (Eq. 1).

288 To evaluate the estimation bias of a parameter ω by using an estimator $\hat{\omega}$, we define a *bias index*
 289 $BI = E(\hat{\omega})/\omega$. The bias $E(\hat{\omega}) - \omega$ is positive if $BI > 1$, zero for $BI = 1$, and negative if $BI < 1$. Here,
 290 to assess the estimation bias for the p -variance of the sample mean, we form:

$$BI = \frac{E_{MC}(\hat{V}_p(\bar{y}_s))}{V_{MC}(\bar{y}_s)} \quad (10)$$

291 where $E_{MC}(\cdot)$ and $V_{MC}(\cdot)$ are the mean and variance calculated over the 10^6 replications of the Monte
 292 Carlo study.

293 4.3.1. Nonpreferential sampling — variable size (BERNWOR)

294 The sampling process model corresponding to (inherently) nonpreferential sampling of variable size
 295 is BERNWOR. The p -distribution of \bar{y}_s for $\pi = 0.1$ ($E_p(n_s) = 500$) is shown in Fig. 4.a.

296 In the BERNWOR case, the p -unbiased estimator of the population mean is the weighted esti-
 297 mator \bar{y}_π , which differs from the sample mean \bar{y}_s . This difference implies that the estimation of the
 298 population mean using the sample mean is p -biased. However, the bias is generally negligible and of no
 299 consequence, as shown in our example (Fig. 4.a). Here, although mathematically biased, the sample
 300 mean is actually a preferable estimator to the weighted estimator (see Särndal et al., 1992, p. 64, Eq.
 301 3.2.6 and Tillé, 2020, p. 35).

302 The default estimator for the p -variance of \bar{y}_s (Eq. 1) is slightly positively biased since we obtain
 303 $BI \approx 1.1136$. The p -variance of \bar{y}_s under the BERNWOR model is approximately the same as that
 304 under the SRSWOR model (Särndal et al., 1992, p. 65) (see Section 4.3.2 and Fig. 4).

305 4.3.2. Nonpreferential sampling — fixed size (SRSWOR)

306 The sampling process model corresponding to the (inherently) nonpreferential fixed-size sampling
 307 is SRSWOR. The p -distribution of \bar{y}_s for $\pi = 0.1$ ($n = 500$) is shown in Fig. 4.b.

308 As recalled in Section 3, in the SRSWOR case, we have the identity $\bar{y}_\pi = \bar{y}_s$. This identity implies
 309 that the estimation of the population mean using the sample mean is p -unbiased (Fig. 4.b).

310 Conversely, the default estimator of the p -variance (Eq. 1) is positively biased since we obtain
 311 $BI \approx 1.1111$. This is because the unbiased estimator in the SRSWOR model (Eq. 7) introduces a
 312 *finite population correction* (see Cochran, 1977, Sec. 2. 6) $fpc = 1 - \pi$, which is closer to 0 the closer
 313 π is to 1. At the extreme, the p -variance becomes zero when $n = N$ (*finite population consistency*;
 314 see Cochran, 1977, Sec. 2.4, p. 21 or Hankin et al., 2019, p. 325). The magnitude of the p -variance
 315 overestimation depends on the value of the sampling fraction $\pi = n/N$, which remains unknown when
 316 N is not known. Let us note that the value obtained by Monte Carlo simulation for BI agrees with
 317 the theoretical value $(1 - \pi)^{-1}$ for $\pi = 0.1$.

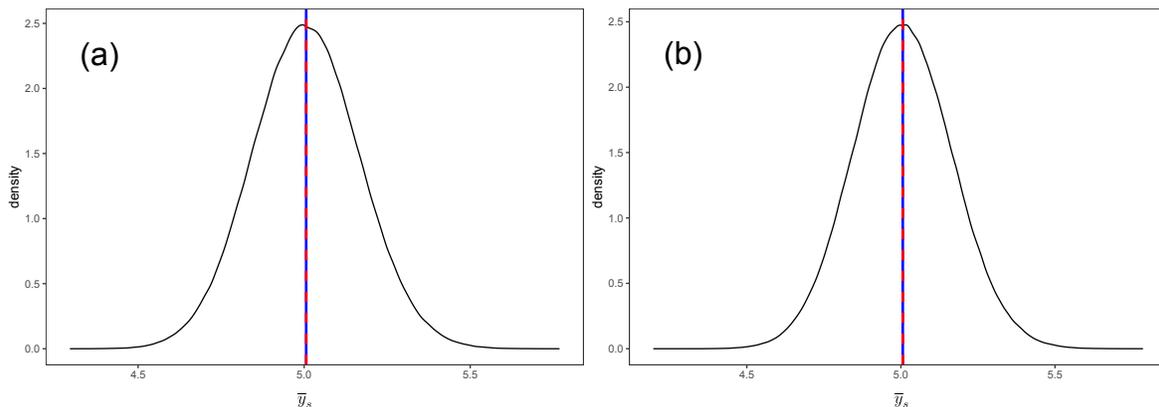


Figure 4: Approximate sampling distributions using 10^6 replications of each nonpreferential sampling model for $\pi = 0.1$. (a) BERNWOR. (b) SRSWOR. The population mean is shown by the red dashed line. The average of the values taken by \bar{y}_s is shown by the blue line.

318 4.3.3. Preferential sampling — variable size (POISSWOR)

319 The sampling process model corresponding to variable-size preferential sampling is POISSWOR.
 320 The vector of probabilities $\boldsymbol{\pi}$ is computed as explained in Section 4.3 for $E_p(n_s) = 500$ (variable sample
 321 size). The p -distributions of \bar{y}_s for $R_{\pi y} = 0.5$ and $R_{\pi y} = 0.9$ are shown in Fig. 5.

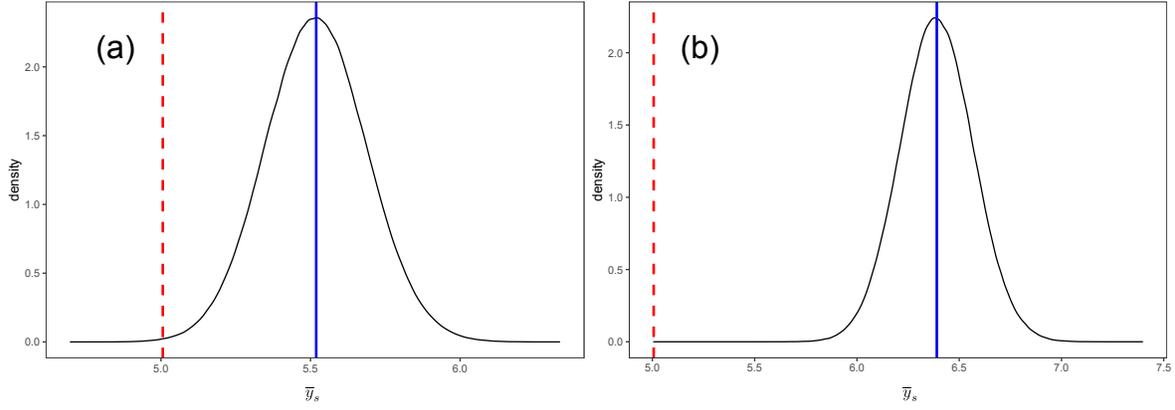


Figure 5: Approximate sampling distributions using 10^6 replications of the POISSWOR model with $P = 500$. (a) $R_{\pi y} = 0.5$. (b) $R_{\pi y} = 0.9$. The population mean is shown by the red dashed line. The average of the values taken by \bar{y}_s is shown by the blue line.

322 In the case of POISSWOR, the p -unbiased estimator of the population mean is the weighted
 323 estimator \bar{y}_{π} , which is different from the sample mean \bar{y}_s . Therefore, like with BERNWOR, the
 324 estimation of the population mean using the sample mean is p -biased. However, unlike the BERNWOR
 325 case, this should not be neglected since the relative p -bias is approximately 10.2% for $R_{\pi y} = 0.5$ and
 326 27.6% for $R_{\pi y} = 0.9$ (Fig. 5). The default estimator for the p -variance of \bar{y}_s (Eq. 1) is positively
 327 biased since we obtain $BI \approx 1.1411$ for $R_{\pi y} = 0.5$ and $BI \approx 1.1788$ for $R_{\pi y} = 0.9$.

328 4.3.4. Preferential sampling — fixed size (CPS)

329 The sampling process model corresponding to fixed-size preferential sampling is CPS. The π -vector
 330 is the same as that for POISSWOR. The p -distributions of \bar{y}_s for $R_{\pi y} = 0.5$ and $R_{\pi y} = 0.9$ are shown
 331 in Fig. 6.

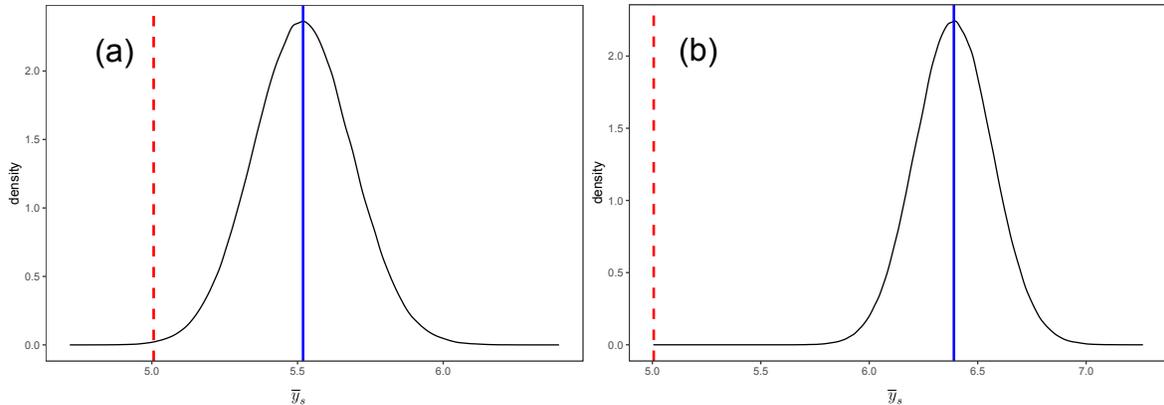


Figure 6: Approximate sampling distributions using 10^6 replications of the CPS model for $P = 500$. (a) $R_{\pi y} = 0.5$. (b) $R_{\pi y} = 0.9$. The population mean is shown by the red dashed line. The average of the values taken by \bar{y}_s is shown by the blue line.

332 In our example, the case of CPS is very similar to that of POISSWOR since the relative p -bias of
 333 the sample mean is approximately 10.2% for $R_{\pi y} = 0.5$ and 27.6% for $R_{\pi y} = 0.9$ (Fig. 6). The default
 334 estimator for the p -variance of \bar{y}_s (Eq. 1) is also positively biased since we obtain $BI \approx 1.1387$ for
 335 $R_{\pi y} = 0.5$ and $BI \approx 1.1793$ for $R_{\pi y} = 0.9$, values that are close to those obtained in the POISSWOR
 336 case.

337 4.3.5. Zero-correlation between the propensity and the variable of interest

338 Thus far, we have simulated preferential sampling for $R_{\pi y} = 0.5$ and $R_{\pi y} = 0.9$. In this section,
 339 we examine the degenerate situation where $R_{\pi y} = 0$.

340 When the correlation between the propensities and y -values is zero, the relative p -bias of the
 341 sample mean is essentially zero for both POISSWOR and CPS (Fig. 7). The default estimator for the
 342 p -variance of \bar{y}_s (Eq. 1) is positively biased, with $BI \approx 1.1223$ for the POISSWOR and $BI \approx 1.1201$ for

343 the CPS. Although the propensities are variable, sampling is nonpreferential (Fig. 7). The situation
 344 is broadly equivalent to that encountered with the BERNWOR and SRSWOR models (Fig. 4).

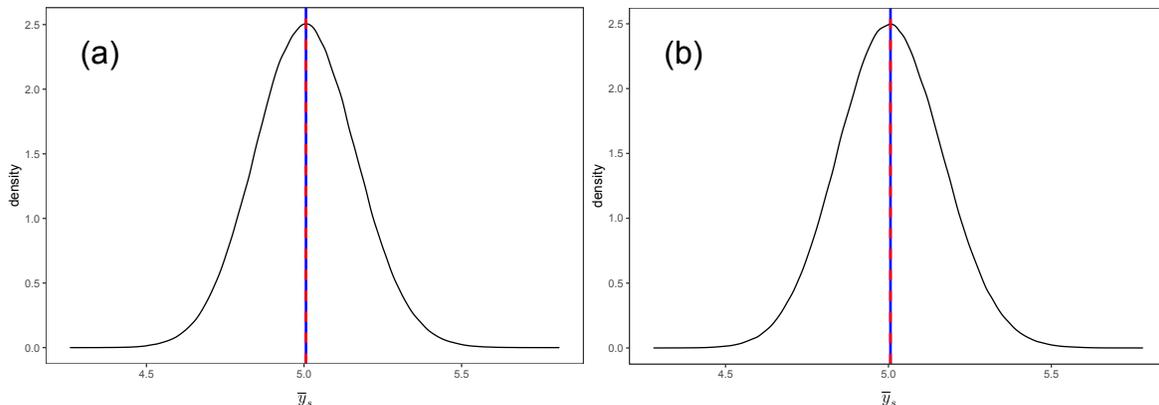


Figure 7: Approximate sampling distributions using 10^6 replications of each unequal propensity sampling model for $P = 500$ and $R_{\pi y} = 0$. (a) POISWOR. (b) CPS. The population mean is shown by the red dashed line. The average of the values taken by \bar{y}_s is shown by the blue line.

345 4.4. An analytical formula for the bias of the sample mean

346 In this section, we focus on the formal expression of the p -bias of the sample mean \bar{y}_s for estimating
 347 the population mean $\bar{y}_{\mathcal{U}}$. For algebraic details, the reader is referred to [Appendix B](#).

348 For a sampling design of variable size, the p -bias of the sample mean can be written as ([Ap-
 349 pendix B.1](#)):

$$B_p(\bar{y}_s) = E_p(\bar{y}_s) - \bar{y}_{\mathcal{U}} \approx \frac{R_{\pi y} S_{\pi} S_y}{\bar{\pi}_{\mathcal{U}}} \quad (11)$$

350 where $R_{\pi y} S_{\pi} S_y = S_{\pi y}$ is the (adjusted) covariance between $\boldsymbol{\pi}$ and \mathbf{y} (see [Appendix A](#)).

351 In our example, we have $\bar{y}_{\mathcal{U}} \approx 5$, $S_y \approx 3.7787$, $N = 5000$ and $\bar{\pi}_{\mathcal{U}} = 0.1$. For $R_{\pi y} = 0.5$, we have
 352 $S_{\pi} \approx 0.0272$, from which we obtain $B_p(\bar{y}_s) \approx 0.51$ and a relative p -bias of approximately 10.2%, in
 353 agreement with what we obtained via Monte Carlo simulation in the POISSWOR case (Section 4.3.3).
 354 For $R_{\pi y} = 0.9$, we have $S_{\pi} \approx 0.0407$, from which we obtain $B_p(\bar{y}_s) \approx 1.38$ and a relative p -bias of
 355 approximately 27.6%, which again matches the value found earlier by Monte Carlo simulation.

356 In the case of a fixed-size sampling design, the p -bias of the sample mean admits an exact expression
 357 (Eq. B.11, [Appendix B.2](#)), but we can also use expression (11) as a close approximation when N is
 358 not too small. Consequently, we obtain the same results for CPS as for POISSWOR.

359 As expected, the observations in the cases studied by Monte Carlo simulation match the results
 360 computed by using formula (11). In the context of Monte Carlo simulation, when we are interested in
 361 the p -bias of the sample mean, this formula allows us to avoid replicating a sampling process model.
 362 We use this computational shortcut when appropriate throughout the remainder of this article.

363 Examination of formula (11) shows that the sample mean is a p -unbiased estimator of the population
 364 mean in the following two situations: (i) when the propensities are variable ($S_{\pi} > 0$), if there is no
 365 correlation between the propensities and the y -values ($R_{\pi y} = 0$) or (ii) when propensities do not vary
 366 ($S_{\pi} = 0$), since then the covariance and correlation are zero ($S_{\pi} = 0 \Rightarrow S_{\pi y} = 0 \Rightarrow R_{\pi y} = 0$). If the
 367 covariance between the propensities and the y -values remains unchanged (fixed $S_{\pi y}$), the lower the
 368 sampling fraction (fixed sample size) or its expectation (variable sample size) $\bar{\pi}_{\mathcal{U}}$ is, the greater the
 369 bias. As expected, the p -bias is zero in the case of a census since the propensities do not vary (and are
 370 all equal to 1), which is the same as in case (ii) above.

371 **5. Trend assessment**

372 For operational purposes, a trend can be defined as a pattern of change in the indicator of interest
 373 over time. In the following, we consider a monitoring program designed to study the trend of a
 374 population parameter over a period Δ . The monitoring program starts at time t_0 and ends at time
 375 $t_0 + \Delta$.

376 *5.1. Trend example*

377 As an example of a trend to be estimated, we consider here the trend in the abundance of a species
 378 over a bounded domain \mathcal{D} of the Euclidean plane, partitioned by a population \mathcal{U} of subdomains d_i
 379 ($i = 1, 2, \dots, N$), formally:

$$\mathcal{D} = \bigcup_{i \in \mathcal{U}} d_i \quad \text{with } d_i \cap d_j = \emptyset \text{ for } i \neq j \in \mathcal{U} \quad \text{and} \quad |\mathcal{D}| = \sum_{i \in \mathcal{U}} |d_i| \quad (12)$$

380 In a general sense, subdomains can have different shapes and sizes. To simplify the presentation, in
 381 what follows, we consider the case where \mathcal{D} is an iso-oriented rectangular domain (i.e., its sides are
 382 parallel to the abscissa and ordinate axes), lying between x_{\min} and x_{\max} on the abscissa and between
 383 y_{\min} and y_{\max} on the ordinate. With such a domain, subdomains can simply be square cells of equal
 384 size. Thus, \mathcal{D} is discretized into a spatial population of $NX \times NY = N$ grid cells of equal size, where
 385 NX and NY are the numbers of columns and rows in the grid, respectively.

386 For the species of interest, at time t (omitted from the notation for simplicity), an individual has
 387 a certain probability density $f(u) > 0$ of being present at a point $u \in \mathcal{D}$ with Cartesian coordinates
 388 (x_u, y_u) . The value of the variable of interest y_i is the number of individuals present in grid cell d_i
 389 ($i \in \mathcal{U}$), i.e., the number of points u where the species is present such that $u \in d_i$ (here, a point
 390 represents only one individual).

391 When the size N of the sampled spatial population \mathcal{U} is known, it is the same, to within a factor
 392 — i.e., N for the point estimator and N^2 for its p -variance — to estimate the mean or the total
 393 abundance. For the concrete situation of preferential sampling that we address in this article (Section
 394 2.4), we consider that N is unknown to the sampler. Thus, in what follows, the parameter of interest
 395 is the population mean $\bar{y}_{\mathcal{U}}$, estimated by the sample mean \bar{y}_s , as in Section 4. We assume that the
 396 spatial variability of the variable of interest is greater on the abscissa than on the ordinate and that
 397 at time t_0 , there is a spatial gradient in abundance from x_{\min} to x_{\max} .

398 *5.2. Population model example*

399 *5.2.1. Simulating an inhomogeneous binomial point process*

400 The spatial distribution of a fixed number of individuals M over \mathcal{D} can be modeled by an inhomogeneous
 401 binomial point process (abbreviated as IBPP) with a spatial intensity function $\lambda(u)$. Since
 402 a binomial point process is a Poisson point process conditional on M , simulation of an IBPP can be
 403 performed as for an inhomogeneous Poisson point process using the Lewis-Shedler method (see, e.g.,
 404 Illian et al., 2008, p. 119; Baddeley et al., 2016, Sec. 5.4.2). An IBPP is simulated as follows:

- 405 1. Generate a point $u \in \mathcal{D}$ following a Bernoulli point process.
- 406 2. The probability of keeping this point is computed as $p(u) = \lambda(u)/\lambda^*$, where λ^* is the maximum
 407 value of the intensity on \mathcal{D} :

$$\lambda^* = \max_{u \in \mathcal{D}} \lambda(u)$$

- 408 3. A Bernoulli trial is performed with probability $p(u)$. If the trial is successful, then the point u is
 409 kept.
- 408 4. Steps 1 to 3 are repeated until M points are drawn.

409 From a realization of the IBPP, the \mathbf{y} -vector is obtained by counting the number of points within
 410 each grid cell d_i ($i \in \mathcal{U}$). Simulating the realizations of the variable of interest directly by allocating
 411 the M individuals among the N grid cells is equivalent to doing so. First, for $i = 1, 2, \dots, N$, the
 412 probability p_i of drawing grid cell d_i is computed as follows (we recall that the grid cells form a
 413 partition of \mathcal{D}):

$$p_i = \frac{\int_{d_i} \lambda(u) du}{\sum_{i \in \mathcal{U}} \int_{d_i} \lambda(u) du} = \frac{\int_{d_i} \lambda(u) du}{\int_{\mathcal{D}} \lambda(u) du} \quad (13)$$

414 In our case, for all $i \in \mathcal{U}$, we have an area $|d_i|$ (i) that is very small compared to area $|\mathcal{D}|$ and (ii)
 415 that is a constant (the square grid cells have the same area). Therefore, in two steps, we obtain the
 416 following approximation:

$$p_i \approx \frac{\lambda(u_i) |d_i|}{\sum_{i \in \mathcal{U}} \lambda(u_i) |d_i|} = \frac{\lambda(u_i)}{\sum_{i \in \mathcal{U}} \lambda(u_i)} \quad (14)$$

417 where u_i is the barycenter of grid cell d_i .

418 The M individuals are allocated as follows:

- 419 1. Initialize $y_i \leftarrow 0$ for $i = 1, 2, \dots, N$.
- 420 2. Select a grid cell of index j by unequal probability sampling with replacement (also known as
 421 *multinomial sampling*) (e.g., [Johnson et al., 1997](#), Sec. 8; [Aubry, 2023](#), Remark 6) with drawing
 422 probabilities p_i ($i = 1, 2, \dots, N$).
- 423 3. Increment $y_j \leftarrow y_j + 1$.
- 424 4. Steps 2 and 3 are repeated until M grid cells are drawn.

425 5.2.2. Spatial intensity function

426 As mentioned above, we assume that the spatial distribution of individuals over \mathcal{D} is governed by
 427 a spatial intensity function $\lambda(x)$ describing a one-dimensional gradient along the x -axis (we now use x
 428 instead of x_u to simplify the notation). The shape of this spatial gradient is determined by the shape
 429 parameter η (Fig. 8) according to the expression:

$$\lambda(x) = \frac{\lambda^* \exp(\eta x')}{\delta} \quad \text{with } x \in [x_{\min}, x_{\max}] \quad (15)$$

430 where $x' \in [0, 1]$ is defined as follows:

$$x' = \frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad (16)$$

431 and

$$\delta = \begin{cases} 1 & \text{if } \eta < 0 \\ \exp(\eta) & \text{otherwise} \end{cases} \quad (17)$$

432 The degenerate case $\lambda(x) = \lambda^*$ is obtained for $\eta = 0$ (Fig. 8) and corresponds to a homogeneous
 433 binomial point process and equiprobable allocation.

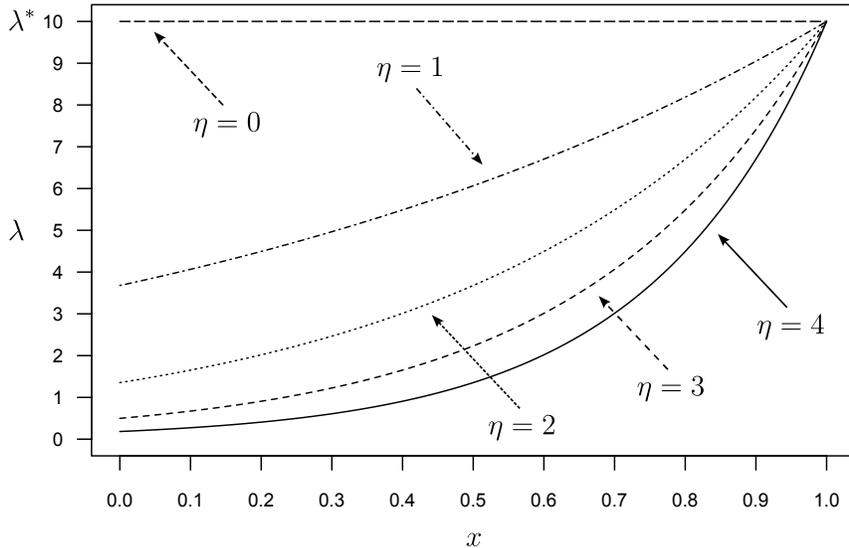


Figure 8: Plot of the intensity function $\lambda(x)$ on the interval $x \in [0, 1]$ with $\lambda^* = 10$, for $\eta = 0, 1, 2, 3, 4$.

434 We assume that the shape of the spatial gradient changes over time in such a way that its direction
 435 is completely reversed at time $t_0 + \Delta$, i.e., it is ultimately from x_{\max} to x_{\min} . This can be achieved with
 436 the shape parameter η (Eq. 15) by gradually decreasing its initial value η_0 at time t_0 until $\eta_\Delta = -\eta_0$
 437 at time $t_0 + \Delta$ (example in Fig. 9).

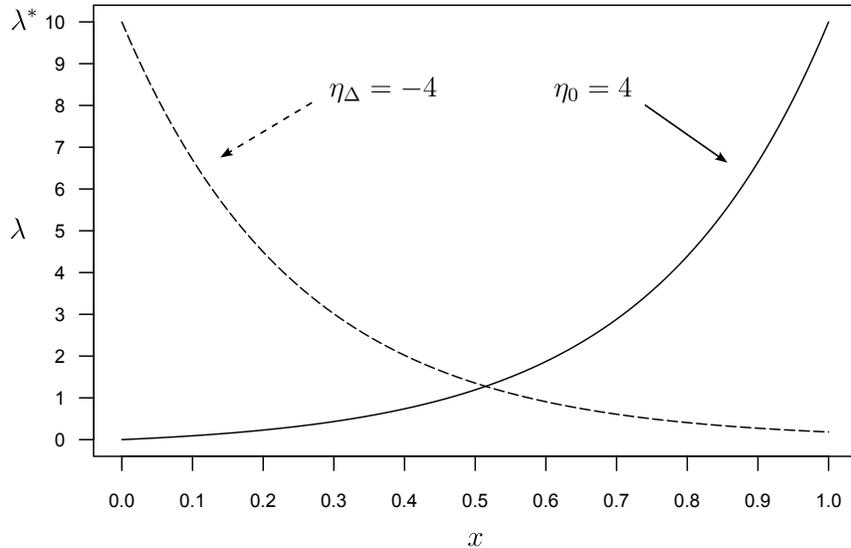


Figure 9: Plot of the intensity function $\lambda(x)$ on the interval $x \in [0, 1]$ with $\lambda^* = 10$, at time t_0 ($\eta_0 = 4$, solid line) and at time $t_0 + \Delta$ ($\eta_\Delta = -4$, dashed line).

438 5.3. Monte Carlo study

439 We assume that the number of individuals M in \mathcal{D} remains constant over the monitoring period
 440 (no demographic change).

441 Preferential sampling of fixed size n is performed according to the sampler's view of the variation
 442 in abundance over \mathcal{D} at time t_0 , favoring the sampling units expected to be the richest in individuals,
 443 i.e., those with the highest y -values. As mentioned in Section 3, this type of preferential sampling
 444 process can be modeled by conditional Poisson sampling (CPS). Under preferential sampling, we have
 445 shown that using the sample mean biases the mean abundance estimation over \mathcal{D} (Section 4). This is
 446 not a problem for assessing trends or changes between two points in time, as long as the bias remains
 447 (approximately) constant over the monitoring period, which here implies that the units that were
 448 expected to be the richest in individuals at time t_0 remain so between t_0 and $t_0 + \Delta$. Conversely,
 449 let us consider a directional change in the spatial distribution of individuals in \mathcal{D} over the monitoring
 450 period. In this situation, using the sample mean to estimate the population mean results in estimating
 451 a spurious trend.

452 In our example, at the start of the monitoring program, the sampler knows that there is a spatial
 453 gradient in abundance from x_{\min} to x_{\max} but does not know its exact shape. By default, we simulate
 454 this situation by using a size variable expressed as $z_i = a \times x_i + b$ with (a, b) such that $z_i > 0$ for all
 455 $i \in \mathcal{U}$ to guarantee a probability $\pi_i > 0$ of being part of the sample.

456 We thus have (i) a fixed vector of propensities ($\boldsymbol{\pi}$) computed from the size variable (\mathbf{z}) (Eq. 8, Fig.
 457 10, top panel) and (ii) a random vector of abundances (\mathbf{y}) obtained according to the intensity function
 458 $\lambda(x)$ (Eq. 15, Fig. 10, bottom panel). For a given realization of the spatial variation model defined by
 459 $\lambda(x)$, the correlation between the propensities and the abundances takes a certain fixed value $R_{\pi y}$. The
 460 model can generate an infinite number of \mathbf{y} -vectors, hence an infinite number of $R_{\pi y}$ -values (within the
 461 range of variation of $R_{\pi y}$ for this model). If the population size is sufficiently large, for convenience,
 462 the correlation between the propensities and the abundances in the model ($\rho_{\pi y}$) can be assimilated to
 463 the ξ -expectation of the population correlation $R_{\pi y}$ (Appendix A, Fig. 10):

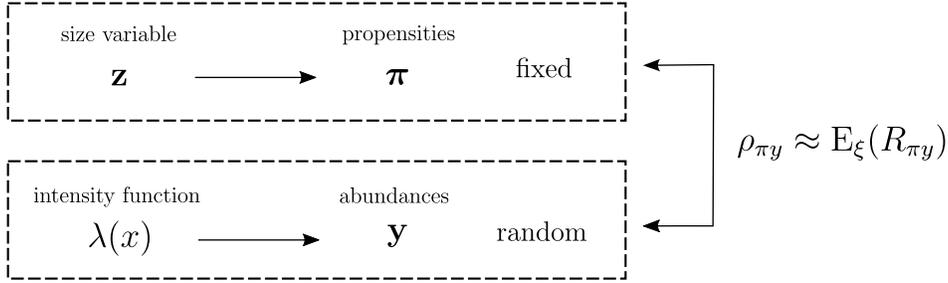


Figure 10: Schematic representation of the relationships between the simulation components. Top panel: The size variable (\mathbf{z}) determines the propensities ($\boldsymbol{\pi}$), which remain fixed. Bottom panel: The intensity function of the inhomogeneous binomial point process ($\lambda(x)$) determines the vector of abundances (\mathbf{y}), which is random. If the population size is sufficiently large, for convenience, the correlation in the model ($\rho_{\pi y}$) can be assimilated to the ξ -expectation of the finite population correlation $R_{\pi y}$.

464 In the following, we first consider the case of a fixed population (i.e., a given realization \mathbf{y}) and then the
 465 case of the superpopulation model itself. We decrease the shape parameter η to simulate the reversal
 466 of the spatial gradient in abundance over \mathcal{D} between t_0 and $t_0 + \Delta$, where we assume that η decreases
 467 linearly with time. For simplicity, we decrease η by one unit per time unit (e.g., per year or decade).
 468 By varying the shape parameter from $\eta_0 = 4$ at time t_0 to $\eta_\Delta = -4$ at time $t_0 + \Delta$ (Fig. 9), we cover a
 469 monitoring period of $\Delta = 8$ time units and have a (short) time series of 9 mean abundance estimates
 470 to fit a (parametric) temporal trend model.

471 We simulate a simple situation where an iso-oriented rectangular domain \mathcal{D} is discretized by $N =$
 472 2500 square cells organized according to a grid of $N_X = 100$ columns and $N_Y = 25$ rows. For
 473 simplicity, we set $x_{\min} = y_{\min} = 0$, $x_{\max} = 100$ and $y_{\max} = 25$ so that each grid cell has a unit area.
 474 The maximum intensity is $\lambda^* = 10$ (the number of points per unit area, i.e., also per grid cell).

475 Regardless of the value of η , $M = 10\,000$ individuals are randomly allocated among the $N = 2\,500$
 476 grid cells, as explained in Section 5.2. Therefore, we have $\bar{y}_{\mathcal{U}} = M/N = 4$. To define the size variable
 477 z , we choose $a = b = 0.1$, that is, $z_i = 0.1 \times x_i + 0.1$ ($i \in \mathcal{U}$). We again use a sampling fraction of
 478 10%, i.e., $n = 250$. The variable propensities that form the $\boldsymbol{\pi}$ -vector result from the previous choices
 479 regarding z and n ; they remain constant throughout the simulations (Fig. 10, top panel).

480 5.3.1. Fixed population

481 Like in Section 4, for each value of η , we keep a \mathbf{y} -vector of abundances for which the correlation
 482 $R_{\pi y}$ is approximately equal to a value of $\rho_{\pi y}$ compatible with the value of η . For each case, we
 483 generate realizations of the \mathbf{y} -vector until we obtain one for which we have $|R_{\pi y} - \rho_{\pi y}| < 10^{-4}$. We
 484 take $\eta = 4, 1, 0, -1, -4$. Compatible correlation values are $\rho_{\pi y} = 0.8, 0.5, 0.0, -0.5, -0.8$.

485 In the case of fixed populations, we are interested in the p -distribution of the sample mean (\bar{y}_s)
 486 and in the p -bias in particular. We replicate 10^6 times the conditional Poisson sampling (CPS) based
 487 on the $\boldsymbol{\pi}$ -vector to accurately approximate the p -distribution of \bar{y}_s . Several samples obtained by CPS
 488 are shown in Fig. 11. As expected for a (putative) linear gradient in abundance from x_{\min} to x_{\max}
 489 and preferential sampling, we see a greater spatial concentration of selected units toward the higher
 490 abscissas and a unit deficit toward the lower abscissas, with a continuum of unit densities in between.

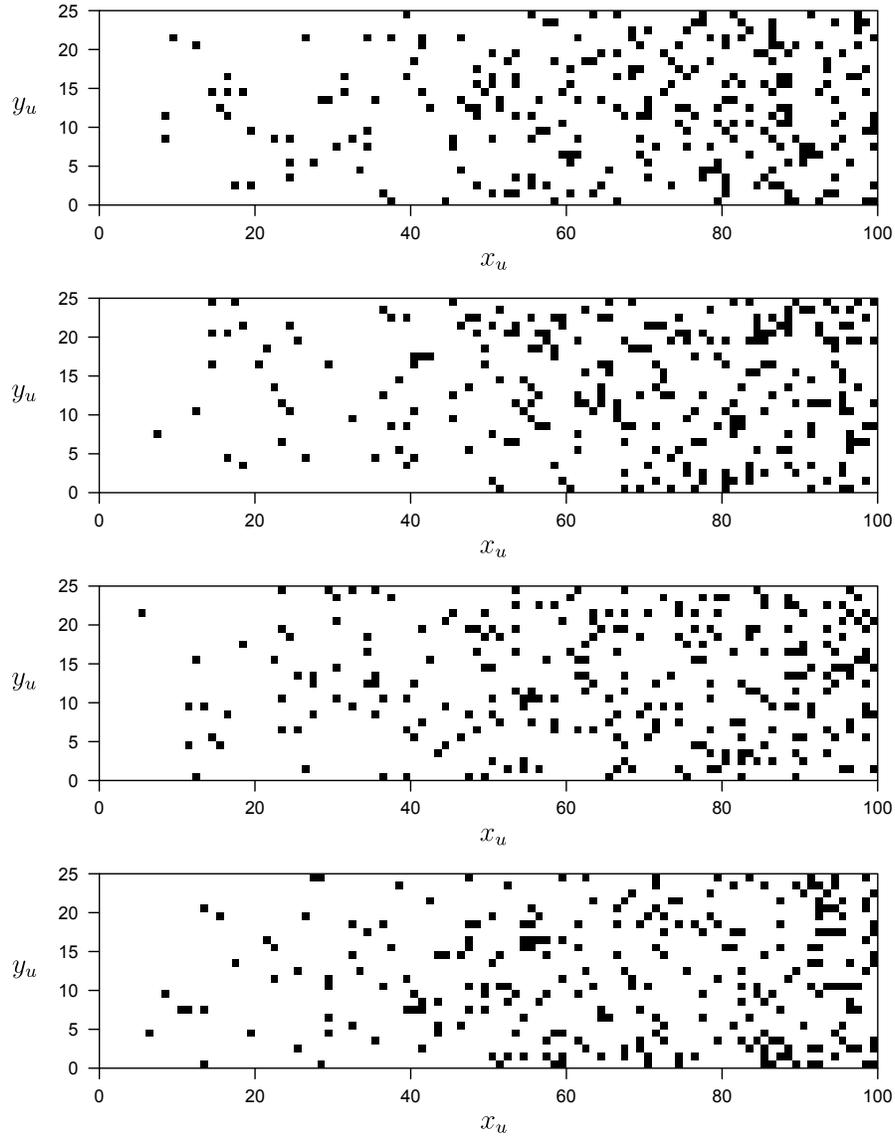


Figure 11: Several samples selected by CPS with inclusion probabilities proportional to a size variable z that varies linearly according to the equation $z_i = 0.1 \times x_i + 0.1$ ($i \in \mathcal{U}$) and for $n = 250$. See the text for details.

491 In agreement with the analytical formula for the bias of the sample mean (Eq. 11), it appears
 492 that the bias is initially positive for $R_{\pi y} > 0$, vanishes for $R_{\pi y} = 0$ and then becomes negative for
 493 $R_{\pi y} < 0$ (Fig. 12). Sampling that planned to be preferential at the beginning of the monitoring
 494 program becomes nonpreferential and eventually antipreferential due to spatial gradient reversal.

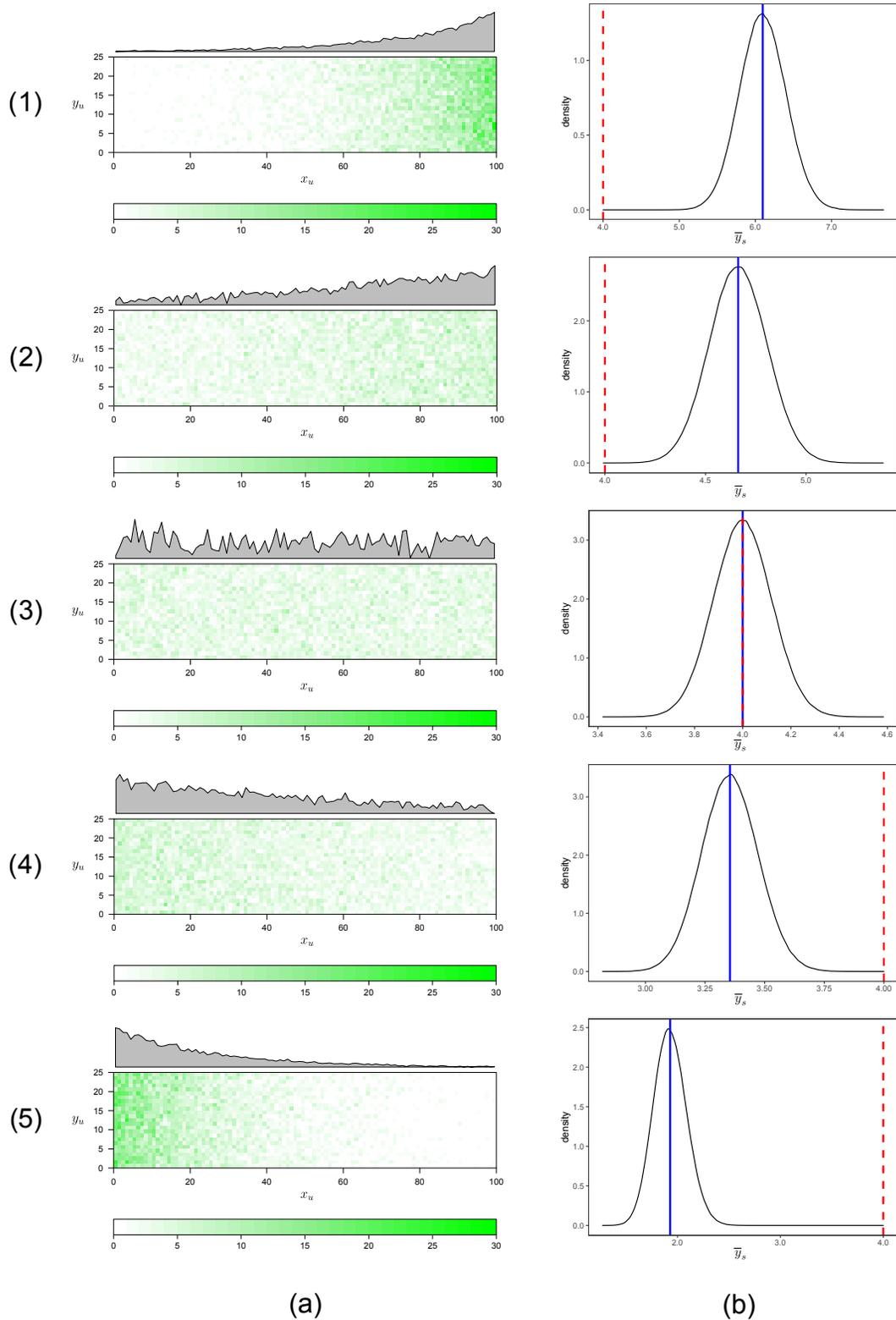


Figure 12: Results of the Monte Carlo study for the five fixed populations. (1) $\eta = 4, R_{\pi y} = 0.8$. (2) $\eta = 1, R_{\pi y} = 0.5$. (3) $\eta = 0, R_{\pi y} = 0$. (4) $\eta = -1, R_{\pi y} = -0.5$. (5) $\eta = -4, R_{\pi y} = -0.8$. (a) Finite populations simulated on a 100×25 grid as a function of the spatial variation model for the set of η -values. The marginal graph (in gray) represents the mean abundance in the grid cells along the x-axis. This reflects the realization of the spatial variation model. (b) Approximations of the p -distributions of the sample mean (\bar{y}_s); 10^6 samples are generated by conditional Poisson sampling. The finite population mean is represented by the red dashed line. The average of the values taken by \bar{y}_s is shown by the blue line. The figures are adjusted to maximize visibility, so the scales may differ from one figure to another. See the text for details.

495 In practice, a sample of spatial units s obtained at time t_0 may remain unchanged over the mon-
 496 itoring period (permanent plots, fixed monitoring network). In the following, we assume that this is
 497 indeed the case. In our example of a reversal spatial gradient, with simulated preferential sampling
 498 and by using the sample mean \bar{y}_s as an estimator of the population mean \bar{y}_U , there is a serious risk
 499 of systematically highlighting a downward trend, while in fact, the abundance in \mathcal{D} remains constant
 500 over the monitoring period (no demographic change).

501 To investigate this hypothesis, at time t_0 , we simulate 10^4 samples using CPS. We compute \bar{y}_s for
 502 each sample and for each of the abundance vectors at the times corresponding to the different values of
 503 η . This gives 10^4 time series of mean abundance estimates. We add four more η values than those used
 504 above, with $\eta = 3, 2, -2, -3$ for which compatible correlation values are $\rho_{\pi y} = 0.79, 0.72, -0.72, -0.79$.
 505 With a time series of 9 mean abundance estimates at hand (for $\eta = 4, 3, 2, 1, 0, -1, -2, -3, -4$), we fit
 506 a linear regression model as a function of time. We use weighted least squares regression (see, e.g.,
 507 [Press et al., 1989](#), Sec. 14.2) to account for the p -variance estimates. For each sample, we thus obtain
 508 a straight line with slope m , and for all the 10^4 samples generated by CPS, we obtain an envelope of
 509 straight lines reflecting the sampling variability of the trend highlighted by the monitoring program.
 510 As expected, in our example, a downward trend is obtained in all the cases, with an average slope of
 511 $E_p(m) \approx -0.56$ (Fig. 13).

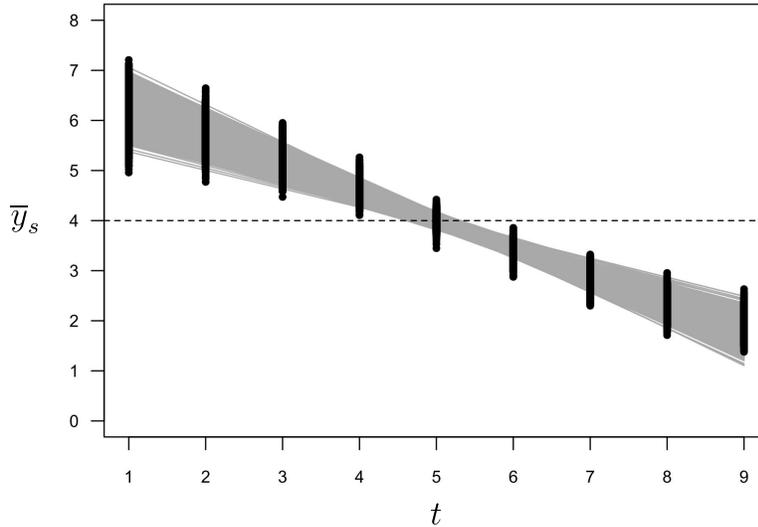


Figure 13: Weighted least squares adjusted linear trend model as a function of time (gray line) for the 9 estimates of the population mean by the sample mean (\bar{y}_s) (black dots). The results for 10^4 samples. See the text for details.

512 5.3.2. Superpopulation

513 In this section, we document the bias of the sample mean \bar{y}_s in the context of both the pref-
 514 erential sampling process modeled by CPS and the process of spatial variation in abundance. The
 515 ξp -expectation of \bar{y}_s is written as follows (we recall that the notation indicates the two sources of
 516 stochasticity involved, with subscript ξ for the superpopulation model and subscript p for the sam-
 517 pling process):

$$E_{\xi p}(\bar{y}_s) = E_{\xi}(E_p(\bar{y}_s)) \quad (18)$$

518 In our model, the population mean \bar{y}_U is constant because we always allocate a fixed number of M
 519 individuals among a fixed number of N units (no demographic change). The ξp -bias is therefore:

$$B_{\xi p}(\bar{y}_s) = E_{\xi p}(\bar{y}_s) - \bar{y}_U \quad (19)$$

520 and the relative ξp -bias is $B_{\xi p}(\bar{y}_s) / \bar{y}_U$. The p -expectation of \bar{y}_s can be expressed by using the p -bias
 521 expression, which gives:

$$B_{\xi p}(\bar{y}_s) = E_{\xi}[B_p(\bar{y}_s) + \bar{y}_U] - \bar{y}_U \quad (20)$$

522 If we are interested only in the value of the ξp -bias (or the relative ξp -bias), as mentioned in Section
 523 4.4, we do not need to replicate the sampling process in our simulation since we know how to compute
 524 the p -bias analytically (Eq. 11 or Eq. B.11). However, it is necessary to simulate \mathbf{y} -vector realizations
 525 to approximate the ξ -expectation that appears in the right term of expression (20).

526 In the following, we simulate 10^4 \mathbf{y} -vectors for each η -value varying between $\eta = 4$ (spatial gradient
 527 in abundance from x_{\min} to x_{\max}) and $\eta = -4$ (gradient from x_{\max} to x_{\min}) with a step of 0.25. We can
 528 also compute the correlation in the model — assimilated here for simplicity to the ξ -expectation of the
 529 population correlation — for each simulated situation. On average, under the model, the population
 530 correlation varies between approximately 0.8 for $\eta = 4$ at time $t = 1$ and -0.8 for $\eta = -4$ at time $t = 9$,
 531 with $\rho_{\pi y} = 0$ for $\eta = 0$ at time $t = 5$ (Fig. 14.a). The relative ξp -bias ranges from approximately
 532 -52.7% for $\eta = -4$ at time $t = 9$ to 52.7% for $\eta = 4$ at time $t = 1$ (Fig. 14.b). As a result,
 533 a preferential sample set up at the beginning of the monitoring program — as explained above —
 534 results in a downward trend (approximately linear) on average under the model, while the number of
 535 individuals did not actually change over the Δ period, but the spatial gradient was reversed.

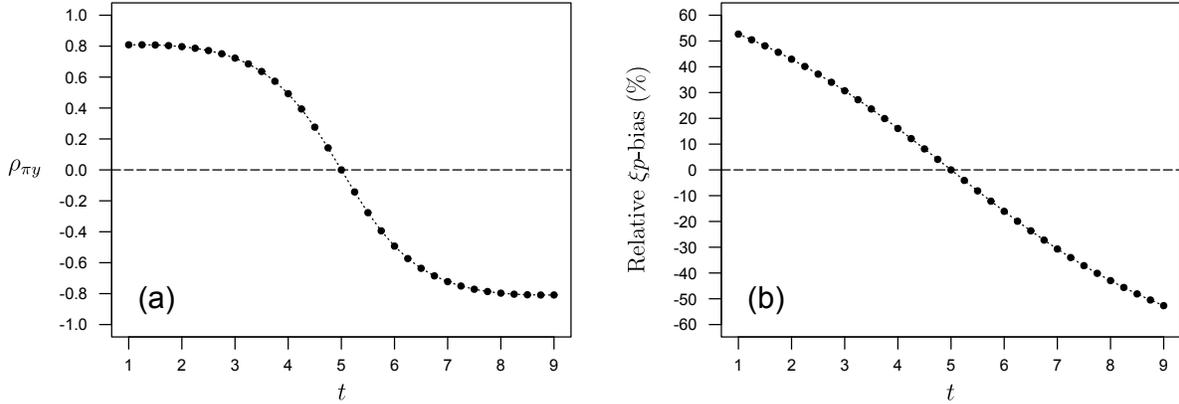


Figure 14: Approximation by Monte Carlo simulation (10^4 realizations of the model) of the relation with time (t) of (a) the ξ -expectation of the population correlation between the propensity and the variable of interest and (b) the relative ξp -bias of the sample mean.

536 6. Discussion

537 To date, the issue of nonprobability preferential sampling (as defined in Section 2.2) has not received
 538 sufficient attention in ecology. This may mean that most ecologists do not have a clear idea of the
 539 implications of this type of nonprobability sampling for their studies and monitoring programs. This
 540 observation led us to examine the issue in detail from a methodological perspective, through the
 541 example of a program aimed at estimating the status or trend of a finite population parameter.

542 First, we formalized the basic instances of without-replacement sampling processes by modeling
 543 them in terms of probability sampling designs. While we have applied this approach to spatial sampling,
 544 its versatility extends to temporal sampling, where the issue of preferential sampling may also arise
 545 (for example, migratory bird surveys are typically conducted at specific times of the year when bird
 546 migration is expected to peak).

547 Next, we used Monte Carlo simulations to study the bias of the sample mean as an estimator of
 548 the population mean, first in the case of status assessment and second in the case of temporal trend
 549 assessment. In the case of estimating the status of a population parameter (e.g., mean species richness),
 550 we illustrated the risk of preferential sampling in terms of bias in the estimation of the population
 551 mean since the relative bias may be unacceptable (e.g., greater than 10%), thereby leading to erroneous
 552 conclusions. In addition, we showed that the estimation of the sampling variance was also biased when
 553 simply using the formula for the variance of the mean s_y^2/n . We also noticed that the fact that the
 554 sampling process results in fixed or variable sample sizes did not appreciably alter the results obtained
 555 and is therefore not in itself a cause for concern. Thus, treating the sample size as fixed and estimating
 556 parameters conditional on its value is a legitimate practice, at least for the question with which we
 557 are concerned in this article, for a sufficiently large population size and moderate sampling fraction.
 558 In the case of estimating the temporal trend of a population parameter such as mean abundance, the
 559 simulated example shows that using preferential sampling at the beginning of the program to establish
 560 permanent plots or sites can lead to the identification of a downward demographic trend when there
 561 is a directional change in the spatial distribution of individuals, even though abundance is actually
 562 constant over the monitoring period.

563 Hence, we clearly demonstrated that, without knowledge of the inclusion probabilities, the sampler
564 lacks awareness of the magnitude of the potential biases in either the sample mean or its sampling
565 variance. The simulated case for the trend assessment is a clear example of *time-varying spatial*
566 *bias* (Johnston et al., 2023, Sec. 3.1). The type of erroneous conclusions reached in this case has
567 already been clearly stated by Fournier et al. (2019) for abundance and Palmer (1993) for community
568 heterogeneity, but these authors referred to the phenomenon of *regression to the mean*, which differs
569 from what we have considered in this article. In a different ecological scenario than the one we
570 considered, McClure and Rolek (2023) noted that preferential sampling may delay the detection of
571 a downward trend. For biological conservation purposes, the common conclusion is that preferential
572 sampling can have a profound impact on trend estimation, either by highlighting a downward trend
573 where none exists (Type I statistical error) or by failing to detect (in time) a downward trend that
574 actually exists (Type II statistical error, insufficient statistical power). These two possibilities should be
575 considered when evaluating the statistical power of a nonprobability preferential sampling monitoring
576 program designed to detect downward trends.

577 In this article, a new perspective on biased site selection was introduced to quantitative ecology.
578 We have modeled preferential sampling processes using probability sampling designs for the purpose
579 of Monte Carlo simulation and also analytical formulation of the estimation bias. For status and trend
580 assessment, we caution against the use of preferential sampling in a nonprobability framework. We
581 showed that the bias in estimating the mean increases with the covariance between the propensity
582 and the variable of interest (species richness, abundance, etc.) and decreases with increasing sampling
583 effort. This is a simple but fundamental result that deserves greater recognition. In the following
584 sections, we take a closer look at the discussion points that emerge from our article.

585 6.1. The scope of the simulated examples

586 The simulated examples in our article are intended to make our findings as clear as possible.
587 Some objections can be raised: (1) real ecological studies and programs are not based on preferential
588 sampling, so the problem documented in this article does not occur in practice; (2) the simulated
589 situations bear no relation to reality.

590 Regarding point (1), it is clear that many programs actually use preferential sampling, at least
591 implicitly. Observers have a general tendency to select sites that are species rich (with rare species)
592 and/or high in abundance rather than opting for sites that may initially seem less interesting for
593 naturalistic observations. This behavior applies to both flora and fauna data collection (see, e.g.,
594 Chytrý, 2001; Lepš and Šmilauer, 2007; Diekmann et al., 2007; Conn et al., 2017; Bowler et al., 2022).
595 Many examples can be provided from the ornithological world, given birds are among the best studied
596 of all animal groups. In the case of Galliforms, for example, a comprehensive review of data sources
597 on a global scale shows that, with the exception of atlases, data tend to be collected preferentially
598 from sites visited by tourists and bird specialists (biodiversity hotspots), while areas with few rare
599 species or protected areas are neglected (Boakes et al., 2010, p. 5). This results in a spatial coverage
600 problem typical of preferential sampling but which also involves convenience sampling because of access
601 difficulties (for logistical or political reasons). Another example is the *Dutch Breeding Bird Monitoring*
602 *Program* (BMP), which again relies on both convenience and preferential sampling since the observers
603 are free to choose their study areas, and in each habitat, they may prefer the most attractive sites,
604 i.e., those that are species-rich and have high bird densities (van Turnhout et al., 2008). An example
605 of a program that relies on both purposive and preferential sampling is the *International Waterbird*
606 *Count* (IWC) — a site-based counting scheme for monitoring waterbird numbers organized at the
607 supranational level by *Wetlands International* — where sites are defined by the judgment of national
608 coordinators and local observers and where decisions about which sites to count are based on their
609 relative importance (Delany, 2010, Sec. 4). These few examples illustrate that programs based on
610 nonprobability sampling of count sites do not fall under a single sampling type but generally involve
611 some degree of preferential sampling.

612 With respect to point (2), when estimating population trends in the context of global warming,
613 *range shifts* have already occurred in many places for different taxa and can be expressed in terms
614 of latitude, longitude, elevation or depth (see, e.g., McCarty, 2001; Parmesan, 2006, 2019; Lenoir
615 and Svenning, 2013, 2015; Dahms and Killen, 2023 and references cited therein). While real-world
616 situations may not be as extreme as the complete reversal of a spatial gradient in abundance as
617 simulated in this paper, the range shift phenomenon is undeniably real; moreover, its frequency is
618 likely to increase with global warming, as temperature and drought (for terrestrial ecosystems) are
619 the most limiting abiotic factors for many species. For birds, for example, range shifts in wintering
620 areas have been well documented (e.g., Maclean et al., 2008, Lehikoinen and Sparks, 2010; Lehikoinen
621 et al., 2013). For some species, this phenomenon may also act in synergy with a change in predation

622 pressure (Pokallus and Pauli, 2015). Changes in trophic interactions may contribute to a change in
623 the geographic distribution of individuals. For example, this may be the case for the Eider Duck
624 (*Somateria mollissima*), for which there is a possible shift from open islands to forested islands. Since
625 the islands monitored are mostly open islands, the result can be the observation of a spurious downward
626 trend (Ekroos et al., 2012, Sec. 4.2). Due to the gregarious behavior of migratory birds (for example)
627 and communication between individuals, it is also possible that such shifts occur quickly.

628 In addition to the examples simulated in this article, as we have previously explained in detail in
629 another context (Aubry et al., 2020, Sec. 5.1), the estimation by the sample mean is biased if the
630 propensities vary significantly and are correlated with the variable of interest (e.g., species richness,
631 abundance). This aligns well with the situation of preferential sampling described in this article. We
632 note that a similar result occurs when estimating the variance of the sampled population, which may
633 be a goal in itself (see Courbois and Urquhart, 2004), a topic not covered in this article.

634 6.2. The bias of the sample mean

635 Beyond the results obtained by Monte Carlo simulation, we formally showed that the bias of the
636 sample mean as an estimator of the population mean can be written essentially as the population
637 covariance between the propensities and y -values divided by the mean propensity, i.e., the sampling
638 fraction in the case of a fixed-size sampling process or its expectation in the case of a variable-size
639 sampling process (Appendix B). Although Aubry et al. (2020) and Boyd et al. (2023) have recently
640 mentioned the key role played by the correlation between sample membership and the variable of
641 interest, to our knowledge, the analytical expression for the bias of the sample mean has so far remained
642 unknown to the ecological audience. In this respect, our paper fills a statistical gap in the field of
643 quantitative ecology.

644 The analytical expression for the bias (or similarly, the relative bias) of the sample mean can also
645 be found in the statistical literature specializing in the treatment of nonresponse (see Kalton and
646 Maligalig, 1991, Eq. 1.3; Särndal et al., 1992, p. 577, Eq. 15.6.3, 15.6. 4; Bethlehem, 1988, Eq.
647 3.5; Bethlehem, 1999, p. 129; Bethlehem, 2002, p. 276; Särndal and Lundström, 2005, p. 92; Brick
648 and Montaquila, 2009, Eq. 4; Bethlehem, 2009, p. 222; Bethlehem et al., 2011, p. 44; Haziza and
649 Beaumont, 2017, Eq. 3.4). An algebraically equivalent formula, but of less pedagogical interest, is
650 given by Groves et al. (2004b, Appendix) (see also Groves et al., 2004a, p. 182). We would like to
651 draw the reader's attention to the fact that this specialized literature is a methodological resource of
652 utmost interest that should be taken into account in quantitative ecology, as was also recently noted
653 by Chadwick et al. (2024, Box 1).

654 To summarize, when propensities vary among the sampling units, for a given spatial population and
655 variable of interest, there are three possible cases where using the sample mean may or may not result
656 in a biased estimation of the population mean. We illustrate these three cases with the help of Fig.
657 15, where the variances of the propensities and y -values do not change between the three situations
658 examined, with the fixed population being that of Fig. 12.1a. If there is a positive correlation between
659 the propensities and y -values (Fig. 15.1a), then the sampling is preferential, and the sample mean bias
660 is positive (overestimation) (Fig. 15.1b). If the correlation is zero, then, although the propensities
661 are variable (Fig. 15.2a), the sampling is nonpreferential, and there is no bias (Fig. 15.2b). If the
662 correlation is negative (Fig. 15.3a), then the sampling is antipreferential, and the bias is negative
663 (underestimation) (Fig. 15.3b).

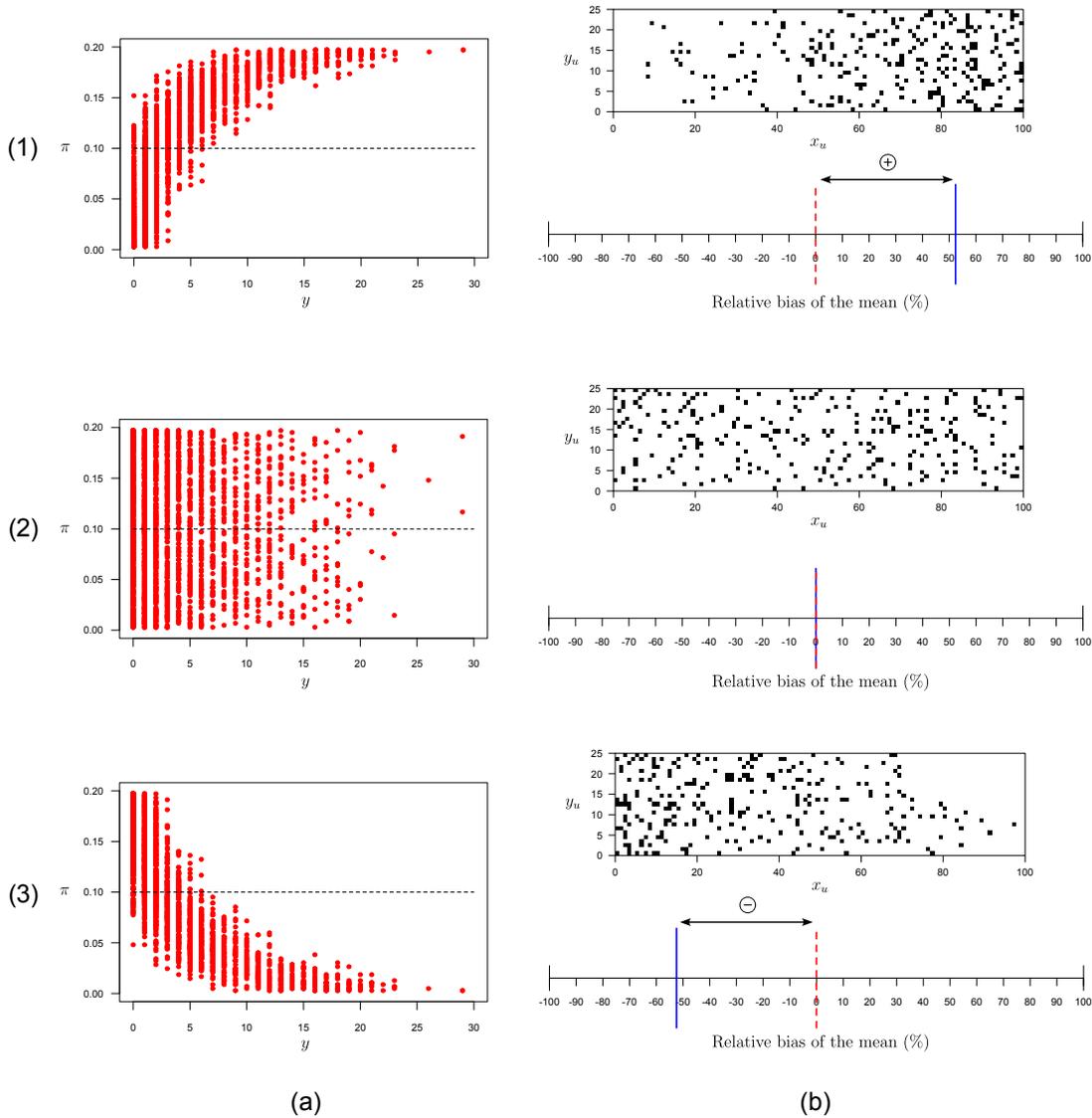


Figure 15: For the fixed population shown in Figure 12.1a, the bias of the sample mean according to the correlation between the propensities and y -values ($R_{\pi y}$). (1) $R_{\pi y} = 0.8$. (2) $R_{\pi y} = 0$. (3) $R_{\pi y} = -0.8$. (a) Scatter plot of the $N = 2500$ sampling units according to their propensity π and y -value. (b) Top panel: Example of a sample drawn by conditional Poisson sampling with the corresponding π -vector. Bottom panel: Relative bias of the sample mean (%) for the corresponding sampling process.

6.3. Reasons for using preferential sampling

Given the risk that preferential sampling poses to the validity of the conclusions drawn from the estimates, we need to ask why it is used. To answer this question, we must first distinguish between two situations, depending on whether the sampler (i) has a sampling frame or (ii) does not.

A sampling frame implies that the size of the spatial population is known. Therefore, it is possible, at least in principle, to use a probability sampling design that allows for the knowledge of inclusion probabilities and their incorporation into the estimators. The sampler can then estimate population parameters within a *design-based* framework that is as objective as possible since there are no assumptions about either the statistical distribution of the variable of interest or its possible spatial structure (spatial trend and autocorrelation), unlike a *model-based* approach (see, e.g., de Gruijter and ter Braak, 1990; Brus and de Gruijter, 1993, 1997; Gregoire, 1998; Dumelle et al., 2022; Aubry and Francesiaz, 2022). In doing so, the sampler is aware of the statistical properties of the chosen estimators. This is a fundamental difference from the concrete situation discussed in this article (see Section 2.4).

There may be several reasons for using preferential sampling within the design-based framework. Usually, the goal is to optimize the efficiency of the sampling strategy — in the sense of a pair formed by a sampling design and an estimator (e.g., Hedayat and Sinha, 1991, p. 24) — by minimizing the p -variance (sampling variance). For a fixed-size sampling design, an examination of the Sen-Yates-Grundy variance formula (e.g., Hedayat and Sinha, 1991, p. 48) — or what leads to the same

682 conclusion, that of its estimator, used in Eq. (6) — shows that the p -variance of the weighted estimator
683 is zero in the case of exact proportionality ($\pi_i \propto y_i$ for $i \in \mathcal{U}$). In practice, we do not know the y -values,
684 but we may know a size variable z which is positively correlated with y . Thus, we approach the ideal
685 proportionality relation by computing the inclusion probabilities as in Eq. (8). For a variable-size
686 sampling design (i.e., Poisson sampling), a similar result holds (see Särndal et al., 1992, pp. 86-87).
687 In a multispecies (i.e., multivariable) context, another reason may be to maximize the probability of
688 covering the spatial distributions of several species, as recently illustrated by Aubry et al. (2023). The
689 goal is to optimize the sampling effort to collect data for a maximum number of species in a limited
690 number of site visits while remaining within the framework of design-based estimation.

691 If the sampler lacks a sampling frame, the selection of the sample of spatial units is typically not
692 randomized — at least as we consider the sampling randomization in this article. Even leaving aside
693 the question of randomization, specifying in advance the spatial units to be visited in such programs
694 can be challenging. In these cases, the preferential nature of the sampling process is then no longer
695 dictated by statistical considerations but rather by concerns about the adherence of the observers —
696 often volunteers — to the program. The preferential nature of the sampling process often results from
697 the collective action of observers who tend to select units in a similar manner (see, e.g., ter Steege
698 et al., 2011).

699 In contrast to preferential sampling, antipreferential sampling may refer to preferential inclusion in
700 the sample of species-poorest units, the units with the lowest abundances or occupancy probabilities.
701 The objectives may be to overrepresent the distribution margins of a species in the sample, to study
702 the effect of buffer zones in the establishment of protected areas or to study the natural recovery of
703 degraded ecosystems.

704 6.4. Reasons for avoiding preferential sampling in trend assessment

705 In general, the use of (i) a permanent sample selected by preferential sampling and (ii) the sample
706 mean as an estimator of the population mean is a sampling strategy that should not be followed for
707 trend assessment. We have illustrated the problem of such a strategy that leads to the conclusion
708 that there is a downward trend even when there is no trend at all. This spurious trend is caused
709 by the concomitance of preferential sampling at the beginning of the program and a change in the
710 geographic distribution of individuals during the monitoring period. In this case, if the propensities
711 (inclusion probabilities) had been known — as would have been the case if the fixed-size sample
712 had been obtained by applying an unequal-probability sampling design such as *conditional Poisson*
713 *sampling* (for example) — then it would have been possible to estimate the mean abundance without
714 bias, as well as the variance of the estimator (see Section 3.5). This would have attenuated the
715 problem of spurious downward trend detection. However, in the simulated case, the sampling variance
716 increases over time as the correlation between the propensities and the y -values decreases, vanishes
717 (null correlation) and then changes sign (negative correlation) (Fig. 14.a). Thus, even if the inclusion
718 probabilities were known and incorporated into the estimators, the strategy used was inappropriate
719 and risky.

720 When assessing a temporal trend in abundance (for example), using a permanent sample of units
721 selected by preferential sampling at the beginning of the monitoring program can potentially lead to
722 three types of erroneous conclusions: (i) highlighting a downward trend when there is no demographic
723 change (the case illustrated in this article); (ii) attenuating an upward trend (e.g., when there is a
724 colonization of new habitat patches that are almost unrepresented in the sample, without changes in
725 habitat patches previously occupied, which constitute the bulk of the sample); and (iii) exaggerating a
726 downward trend (e.g., in the case of a density-dependent decline, more pronounced in units where there
727 were many individuals initially, which represents the bulk of the sample). In the case of antipreferential
728 sampling, the results are reversed. The only case for which this sampling strategy does not lead to a
729 time-varying spatial bias is when the change in abundance is uniform across the spatial domain under
730 consideration, i.e., when it occurs at the same rate for all sampling units. It is understandable that
731 this case is the exception rather than the rule in ecology.

732 6.5. Consideration of the sampling process in ecology

733 Like others (e.g., Elzinga et al., 2001, p. 116; Albert et al., 2010; Smith et al., 2017; Aubry et al.,
734 2020; Boyd et al., 2023), we recommend the use of probability sampling whenever possible to control
735 for inclusion probabilities. This recommendation applies whether the planned inference is design-based
736 or model-based. It is often thought that the use of a probability sampling design is not applicable
737 in ecology, especially on a large scale. While we cannot deny that using probability sampling may
738 indeed be difficult or even unfeasible (see, e.g., Roleček et al., 2007), there are counterexamples that

739 illustrate that this approach may be possible, at least for certain taxonomic groups and environments
740 (e.g., Pavlacky et al., 2017; van Wilgenburg et al., 2020; Aubry et al., 2023).

741 When it is not possible to control for inclusion probabilities, it is important to be aware of the
742 source of bias that preferential sampling may represent and to discuss it when communicating the
743 results of a study or program. We agree with Boyd et al. (2023) on this point while recognizing that
744 this is the least one can do. A further step forward is the recognition that statistically sound use of
745 the data obtained by preferential sampling necessarily requires consideration of the sampling process,
746 whether the chosen inferential framework is frequentist or Bayesian. This topic is an active area of
747 research in survey sampling and spatial or ecological statistics, which goes beyond status and trend
748 assessment (see Journel, 1994; Diggle et al., 2010; Pati et al., 2011; Zidek et al., 2014; Pan et al., 2015;
749 Grisotto et al., 2016; Cecconi et al., 2016; Conn et al., 2017; Irvine et al., 2018; Watson et al., 2019;
750 Dinsdale and Salibian-Barrera, 2019; Pennino et al., 2019; da Silva Ferreira, 2020; Olea, 2021; Gray
751 and Evangelou, 2023; Vedensky et al., 2023). Such research is critical for existing ecological programs
752 that were not built on the basis of probability sampling, but addressing the statistical approaches that
753 can be considered *a posteriori* to deal with the problem of bias induced by preferential sampling is
754 beyond the scope of this article. Nevertheless, for a recent introduction of this topic in ecology, we
755 refer the reader to Boyd et al. (2024).

756 Regarding sampling processes in ecology, the main message of this article can be summarized as
757 follows: "If we do not know the sampling process, we have no idea what we are doing with the data (at
758 least from a statistical perspective)". While this statement may seem self-evident to statisticians, an
759 examination of the ecological literature reveals a disturbing reality: the somewhat widespread neglect
760 of this basic premise. In practice, statistical analysis and modeling are often performed without due
761 consideration of the sampling process. This is not a problem if one is interested only in the data
762 from the sample at hand, but it is a guarantee of poor statistical inference when the results must be
763 extended beyond the sample. We believe that the oversight about the sampling process at work is a
764 major contributor to the misuse of statistical methods in ecology (*analytical crisis*, Chadwick et al.,
765 2024).

766 6.6. Focusing on the terminology

767 Scientific discourse requires accurate, monosemous and, ideally, stable and shared terminology (see,
768 e.g., Schuster, 2020, Sec. 2). Unfortunately, there is no consensus on the terminology used to describe
769 preferential sampling or related concepts. Since the lack of common understanding and naming of
770 concepts is a major barrier to communication among researchers, we felt it important to devote part
771 of the discussion in this article to this issue, to ensure proper and operational transfer in quantitative
772 ecology. We fully agree with Hall et al. (1997) that "[...] if we want to advance [...] ecology, we must be
773 sure that the fundamental concepts with which we work are well defined, and hence, well understood".

774 The term *preferential sampling* has been used in the literature to denote spatial sampling when
775 it is neither regularly nor randomly distributed across the study area (e.g., Goovaerts, 1997, Sec.
776 4.1.1), as a synonym for *purposive sampling* of typical units (e.g., Orlóci, 1975, pp. 10, 12; Podani,
777 1984; Roleček et al., 2007; Swacha et al., 2017), for *convenience sampling*, mainly for ease of access
778 (e.g., Clifford et al., 2011; Mentges et al., 2021). Other distinct meanings can also be found in the
779 literature. For example, Merckx et al. (2011) use the term *preferential sampling* to refer to visiting
780 some sites more frequently than others. In numerous articles, the term is not even defined by the
781 authors who use it, which is problematic because it can mean different things to different people.
782 Moreover, it covers different aspects that need to be distinguished because they may have different
783 statistical consequences, requiring different statistical approaches to be properly handled. In this
784 article, we use the term *preferential sampling* in a precise statistical sense to denote the existence of
785 a nonnegligible correlation between the propensities of the units to be sampled and the values of the
786 variable of interest. It is used in a similar way as McClure and Rolek (2023), except that our definition
787 is more deeply rooted in statistics.

788 Preferential (or antipreferential) sampling is a special case of what is more generally known in the
789 literature as *biased selection* in the sense that the sampling process produces samples in which some
790 parts of the population are underrepresented while others are overrepresented (Zarkovich, 1966, p. 75).
791 The term *selection bias* may be used by some authors to refer to the same idea (e.g., Eklund, 1959, Sec.
792 3.2), in the sense of samples or data biased by selection (*sample selection bias* or *selection-biased data*).
793 This terminology refers to one of the usual meanings given to the term *bias*, i.e., that associated with a
794 distortion or deformation (i.e., of a study, a result, a conclusion, etc.). However, the expression *selection*
795 *bias* can also refer to the technical statistical meaning of the term *bias* when it concerns an estimator
796 (e.g., Kotz et al., 2006, p. 483). We note that biased selection does not necessarily lead to biased
797 estimation (e.g., Overton and Stehman, 1995, Example 6; Aubry et al., 2020, Sec. 5.1), for example,

798 when using a weighted estimator based on inclusion probabilities (e.g., using the Horvitz-Thompson
799 estimator). Conversely, the absence of biased selection does not guarantee unbiased estimation (e.g.,
800 [Stuart, 1984](#), Sec. 6). This terminology can therefore be confusing; the most important thing is to
801 know what one is talking about and to use the terms consistently.

802 The term *site-selection bias* (e.g., [Irvine et al., 2018](#); [Fournier et al., 2019](#); [Mentges et al., 2021](#))
803 is another synonym for *preferential sampling* in the meaning used in this article, or includes it as a
804 subcase ([McClure and Rolek, 2023](#)). The two synonymous terms *response biased sampling* or *response*
805 *selective sampling* can also be found in the literature to refer to (or include) preferential sampling (e.g.,
806 [Lawless, 1997](#); [Lawless et al., 1999](#); [Scott and Wild, 2011](#)). However, these two terms seem to be used
807 almost exclusively by (some) statisticians.

808 By using statistical terminology from the field of missing data (see, e.g., [Allison, 2002](#); [Enders,](#)
809 [2010](#); [Molenberghs et al., 2015](#); [Little and Rubin, 2019](#)), whenever sampling is preferential (Fig. 15.1)
810 or antipreferential (Fig. 15.3), the missingness mechanism is said to be *nonignorable*, and the data are
811 said to be *missing not at random* (MNAR; also referred to as *not missing at random* or NMAR, see
812 [Little and Rubin, 2019](#), p. 28, Note 1). If the propensities vary minimally or vary significantly but
813 are not correlated with the variable of interest (Fig. 15.2), then the mechanism is said to be *ignorable*,
814 and the data are said to be *missing completely at random* (MCAR).

815 [Boyd et al. \(2023, 2024\)](#) use the term *representative* to refer to nonpreferential sampling, i.e.,
816 when sample membership is uncorrelated with the variable of interest. Given the already widely
817 polysemous nature of the terms *representative sampling* and *representative sample* (see, e.g., [Kruskal](#)
818 [and Mosteller, 2006](#); [Bethlehem, 2009](#), Sec. 2.4.1), this new definition is likely to create more confusion
819 than clarification. Since sampling is a mechanism that produces missing data, it is more appropriate
820 to use the terminology used in that field and simply refer to sampling as *ignorable* or *nonignorable*,
821 as the case may be. When dealing with statistical issues in ecology, it is appropriate for clarity and
822 consistency to refer to the vocabulary used in statistics, as [Irvine et al. \(2018\)](#) do, for example.

823 7. Perspectives

824 The points discussed in this article relate to spatial sampling in ecology — that is, sampling of
825 spatial units to study or monitor ecological phenomena — but they are quite general from a statistical
826 perspective and concern broader topics than those covered here (e.g., [Aubry et al., 2020](#)). We have used
827 the (seemingly simple) example of status and trend assessment, but the issue of spatial preferential
828 sampling has also been highlighted in other fields (e.g., spatial prediction, [Gelfand et al., 2012](#)). We
829 have considered preferential spatial sampling — which is undoubtedly the most obvious instance of
830 preferential sampling in ecology — but temporal sampling should also be accounted for in practice.
831 For example, if the sampling period targets an annual peak in abundance but the phenology of the
832 species of interest is gradually changing with global warming, then there is a serious risk of introducing
833 a time-varying temporal bias, which is the counterpart of the time-varying spatial bias illustrated in
834 this article.

835 We believe that the analytical formula governing the bias of the sample mean (Eq. 11) is funda-
836 mental and should be familiar to ecologists and wildlife biologists. Despite the recognized importance
837 of sampling, it is paradoxical that it receives so little attention overall in quantitative ecology, a field
838 largely dominated by modeling. Even in the case of a model-based approach, sampling issues remain
839 central, as they largely determine the ability of the model fitted to the sample data to reliably estimate
840 or predict the quantities of interest. Further attention and work are needed in this area, as we believe
841 this topic is critical to the credibility of the results published in the ecological literature.

842 Demonstrating the risk of the sampling strategy documented in this article for trend assessment
843 is undoubtedly useful, but it is even more useful to suggest strategies that are as robust as possible
844 for detecting a trend that is not spurious, not attenuated or not exaggerated to guide the readers in
845 their choices when designing a monitoring program. It is not simply a matter of drawing inspiration
846 from existing programs but of justifying strategies in light of ecological, statistical and operational
847 perspectives. Quoting [Greenwood \(2003\)](#): *"In designing surveys, however, we strike the balance be-*
848 *tween theoretical robustness and practicality: just as there is no point in running a survey so biased*
849 *that the data are uninterpretable, there is no point in designing one that is so theoretically perfect that*
850 *it is impossible to conduct."* This may be the subject of future articles.

851 Another perspective concerns the situation in which probability sampling is ruled out (for various
852 reasons). A statistical approach for dealing with nonprobability sampling data is to use a model, be it
853 frequentist or Bayesian. This approach is known as the *model-based* approach. Strictly speaking, the
854 *design-based* approach cannot be used, precisely because the sampling was not conducted by using a
855 probability sampling design. However, a remaining question arises: Just as we have used probability

856 sampling designs to model sampling processes for simulation purposes, to what extent can they also
857 be used for statistical inference purposes? This approach differs from design-based inference in the
858 strictest sense of the term because, in this case, the inference is actually based on modeling the sampling
859 process using a probability sampling design. This approach may be called *quasi-randomization* in the
860 sense given by [Oh and Scheuren \(1983\)](#) or *pseudo design-based* ([Baker et al., 2013](#)). This rather
861 unusual topic in quantitative ecology (see [Boyd et al., 2024](#) for a primer) should be explored in detail
862 in future articles and echoes the question posed by [Boyd et al. \(2023\)](#) ” *What other methods for making
863 inferences from nonprobability samples exist, and how reliable are they with real data?*”. Such a study
864 can be undertaken concurrently with an examination of the robustness of predictions based on a
865 superpopulation model, a topic recently illustrated by [Aubry and Francesiaz \(2022\)](#).

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870 Appendix A. Notational conventions

871 We denote the mean of a variable x on a finite set A :

$$\bar{x}_A = \frac{1}{|A|} \sum_{i \in A} x_i$$

872 where $|A|$ is the size (cardinality) of A .

873 We denote the *adjusted variance* of x on the finite population \mathcal{U} :

$$S_x^2 = \frac{1}{N-1} \sum_{i \in \mathcal{U}} (x_i - \bar{x}_{\mathcal{U}})^2$$

874 With this notation, the population variance has $N-1$ as the denominator, following the convention
875 used in survey sampling theory (see [Cochran, 1977](#), p. 23). When the finite population is viewed as
876 randomly drawn from a superpopulation, using $N-1$ as the denominator also makes sense since the
877 population variance is then an unbiased estimator of the superpopulation variance (e.g., [O’Neill, 2014](#),
878 p. 283). Similarly, the (adjusted) sample variance has the denominator $n_s - 1$:

$$s_x^2 = \frac{1}{n_s - 1} \sum_{i \in s} (x_i - \bar{x}_s)^2$$

879 where n_s is the sample size, which may or may not depend on the realized sample s (variable or fixed
880 sample size).

881 We denote the (adjusted) covariance between two variables x and y on the finite population \mathcal{U} :

$$S_{xy} = \frac{1}{N-1} \sum_{i \in \mathcal{U}} (x_i - \bar{x}_{\mathcal{U}})(y_i - \bar{y}_{\mathcal{U}})$$

882 The correlation between two variables x and y on the finite population \mathcal{U} is defined as $R_{xy} =$
883 $S_{xy} / [S_x S_y]$.

884 In a superpopulation model denoted ξ , the ξ -variance of a variable x is noted σ_x^2 , and the ξ -
885 covariance between two variables x and y is denoted σ_{xy} . The ξ -correlation between x and y is then
886 defined as $\rho_{xy} = \sigma_{xy} / [\sigma_x \sigma_y]$. This is the definition used in Section 4. Mathematically, R_{xy} is not an
887 unbiased estimator of ρ_{xy} ; that is, $\rho_{xy} \neq E_{\xi}(R_{xy})$. However, the bias is negligible in the case of a
888 sufficiently large population. In Section 5, for convenience, we use the approximation $\rho_{xy} \approx E_{\xi}(R_{xy})$.

889 Appendix B. Bias of the sample mean

890 Let a finite population \mathcal{U} of size N be sampled by a without-replacement sampling design with
891 inclusion probabilities $0 < \pi_k \leq 1$ ($k \in \mathcal{U}$). Let s be a sample of size n_s drawn from \mathcal{U} . Introducing
892 the indicator variable $I_k = 1$ when unit $k \in \mathcal{U}$ is included in sample s and $I_k = 0$ otherwise, the sample
893 mean is written:

$$\bar{y}_s = \frac{1}{n_s} \sum_{k \in s} y_k = \frac{1}{n_s} \sum_{k \in \mathcal{U}} I_k y_k = \frac{\sum_{k \in \mathcal{U}} I_k y_k}{\sum_{k \in \mathcal{U}} I_k} \quad (\text{B.1})$$

894 *Appendix B.1. Variable-size sampling design*

895 In a variable-size sampling design, the sample size n_s is a random variable of expectation:

$$\mathbb{E}_p(n_s) = \mathbb{E}_p\left(\sum_{k \in \mathcal{U}} I_k\right) = \sum_{k \in \mathcal{U}} \mathbb{E}_p(I_k) = \sum_{k \in \mathcal{U}} \pi_k \quad (\text{B.2})$$

896 By using the exact expression for the expectation of a ratio of two random variables (see [Midzuno, 1950](#); [Koop, 1951, 1972](#)), the expectation of the sample mean (B.1) can be written as follows:

$$\mathbb{E}_p(\bar{y}_s) = \mathbb{E}_p\left(\frac{\sum_{k \in \mathcal{U}} I_k y_k}{\sum_{k \in \mathcal{U}} I_k}\right) = \frac{\mathbb{E}_p\left(\sum_{k \in \mathcal{U}} I_k y_k\right)}{\mathbb{E}_p\left(\sum_{k \in \mathcal{U}} I_k\right)} + \epsilon \quad (\text{B.3})$$

898 with

$$\epsilon = \mathbb{E}_p\left[\left(\sum_{k \in \mathcal{U}} I_k y_k\right) \left\{ \left(\sum_{k \in \mathcal{U}} I_k\right)^{-1} - \left(\sum_{k \in \mathcal{U}} \pi_k\right)^{-1} \right\}\right] \quad (\text{B.4})$$

899 The ϵ -term is usually negligible, but this is not the case for very small populations. Therefore, from
900 Eq. (B.3), we can use the following approximation:

$$\mathbb{E}_p(\bar{y}_s) \approx \frac{\mathbb{E}_p\left(\sum_{k \in \mathcal{U}} I_k y_k\right)}{\mathbb{E}_p\left(\sum_{k \in \mathcal{U}} I_k\right)} = \frac{\sum_{k \in \mathcal{U}} \pi_k y_k}{\sum_{k \in \mathcal{U}} \pi_k} = \frac{1}{N \bar{\pi}_{\mathcal{U}}} \sum_{k \in \mathcal{U}} \pi_k y_k \quad (\text{B.5})$$

901 with an average inclusion probability of $\bar{\pi}_{\mathcal{U}} = N^{-1} \sum_{k \in \mathcal{U}} \pi_k$. Therefore, the bias is approximated as
902 follows:

$$\text{B}_p(\bar{y}_s) = \mathbb{E}_p(\bar{y}_s) - \bar{y}_{\mathcal{U}} \approx \frac{1}{N \bar{\pi}_{\mathcal{U}}} \sum_{k \in \mathcal{U}} \pi_k y_k - \bar{y}_{\mathcal{U}} = \frac{1}{\bar{\pi}_{\mathcal{U}}} \left[\frac{1}{N} \sum_{k \in \mathcal{U}} \pi_k y_k - \bar{\pi}_{\mathcal{U}} \bar{y}_{\mathcal{U}} \right] = \frac{1}{\bar{\pi}_{\mathcal{U}}} \text{Cov}_{\pi y} \quad (\text{B.6})$$

903 with $\text{Cov}_{\pi y} = (N - 1) S_{\pi y} / N$ where $S_{\pi y}$ is the adjusted covariance ([Appendix A](#)). Therefore, we can
904 also write:

$$\text{B}_p(\bar{y}_s) \approx \frac{\text{Cov}_{\pi y}}{\bar{\pi}_{\mathcal{U}}} = \frac{(N - 1) R_{\pi y} S_{\pi} S_y}{N \bar{\pi}_{\mathcal{U}}} \quad (\text{B.7})$$

905 For $(N - 1) / N \approx 1$, we obtain the approximate expression:

$$\text{B}_p(\bar{y}_s) \approx \frac{R_{\pi y} S_{\pi} S_y}{\bar{\pi}_{\mathcal{U}}} \quad (\text{B.8})$$

906 *Appendix B.2. Fixed-size sampling design*

907 With a fixed-size sampling design, the sample size is the constant $n = \sum_{k \in \mathcal{U}} \pi_k$, and the sample
908 mean (B.1) can therefore be written as follows:

$$\bar{y}_s = \frac{\sum_{k \in \mathcal{U}} I_k y_k}{\sum_{k \in \mathcal{U}} \pi_k} \quad (\text{B.9})$$

909 The expectation of the sample mean is then:

$$E_p(\bar{y}_s) = \frac{E_p\left(\sum_{k \in \mathcal{U}} I_k y_k\right)}{\sum_{k \in \mathcal{U}} \pi_k} = \frac{\sum_{k \in \mathcal{U}} \pi_k y_k}{\sum_{k \in \mathcal{U}} \pi_k} = \frac{1}{N\bar{\pi}_{\mathcal{U}}} \sum_{k \in \mathcal{U}} \pi_k y_k \quad (\text{B.10})$$

910 The bias is therefore written exactly:

$$B_p(\bar{y}_s) = \frac{\text{Cov}_{\pi y}}{\bar{\pi}_{\mathcal{U}}} = \frac{(N-1)}{N} \frac{R_{\pi y} S_{\pi} S_y}{\bar{\pi}_{\mathcal{U}}} \quad (\text{B.11})$$

911 For $(N-1)/N \approx 1$, we again obtain the approximate expression (B.8).

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